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MHD ROTATING FLUID FLOW PAST A VERTICAL PLATE THAT APPLIES ARBITRARY SHEAR STRESS WITH RAMPED WALL TEMPERATURE

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A B S T R A C T

Heat transfer in rotating, incompressible viscous fluid near an infinite vertical plate that applies a time-dependent shear stress $f(t)$ with ramped wall temperature is investigated. Closed form exact solutions of the dimensionless governing equations along with imposed initial and boundary conditions are determined using the Laplace transform technique. These solutions are uncommon in literature and can generate exact solutions for any motion of problem with technical relevance of this type. The effects of different system parameters, such as Ekman number, Grashof number, Prandtl number, magnetic number and time on the velocity is examined in detail. Its influence on the fluid motion is graphically displayed. Some special cases together with particular cases are considered.

1. Introduction

The wall conditions about velocity and temperature are arbitrary and non-uniform in many daily life problems. To investigate such problems, the step change in wall temperature is useful. In several practical situations, for example, nuclear heat transfer control, materials processing, building heat transfer, turbine blade heat transfer and electronic circuits, the ramped wall temperature has useful applications. Having such motivation, Narahari *et al*. [1] considered the mass transfer and free convection current effects on unsteady viscous flow with ramped wall temperature. Seth and Ansari [2] investigated the thermal diffusion and heat absorption effects on the MHD natural convection flow past an impulsively started vertical plate with ramped wall temperature. Seth *et al.* [3] studied the impulsive motion of a plate in the presence of radiation effect and considered the fluid to be electrically conducted under the assumption of small magnetic Reynolds' number and passing through a porous medium. Chandran *et al*. [4] investigated analytically the unsteady natural convection flow of an incompressible viscous fluid near a vertical plate with ramped wall temperature using the Laplace transform technique. Sami *et al*. [5] extended this work by investigating the unsteady magneto hydrodynamic flow past an impulsively started vertical plate embedded in a porous medium in the presence of thermal diffusion and ramped wall temperature at the plate. Khan *et al*. [6] investigated the radiation and thermal diffusion/Soret effects on a magneto

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hydrodynamic (MHD) free convection flow of an incompressible viscous fluid near an oscillating plate embedded in a porous medium.

Besides the ramped wall temperature, rotating flows make an important branch of fluid dynamics. Because, in many practical applications, the thermal rotating flows occur in a variety of rotating machinery. Together with heat transfer, the rotating flows applications are found in some natural phenomena such as geophysical systems, tornadoes, hurricanes and ocean circulations. The flow and heat transfer due to moving surfaces have many practical applications, such as in polymer processing systems, production of paper and insulating material. Khan *et al*. [7, 8] presented the magneto hydrodynamic rotating flow of a generalized Burgers'fluid in a porous medium. However, they only considered the momentum transfer and the heat transfer phenomenon was not incorporated. Perhaps, it is due to the fact that when momentum equation becomes coupled with energy equation because of free convection term, then the resulting equation becomes complicated and it is not easy to solve particularly in closed form. Therefore, mostly the rotating flow problems are dealt with numerical or approximate techniques in such types of situations [9, 10]. Ismail *et al*. [11, 12] investigated rotation effects on unsteady MHD free convection flow in a porous medium followed by Qushairi *et al*. [13] where they studied the unsteady free convection flow of a rotating second grade passing through a porous medium. However, it is worth pointing out that all these papers have a common specific feature. Namely, they solved problems in which the velocity is given on the boundary. This constitutes one of the three types of boundary value problems in fluid mechanics. In other two types, either shear stress is given on the boundary or velocity and shear stress are mixed at the boundary wall. From theoretical and practical point of view, all three types of boundary conditions are identically important. In the last two years, the researchers working on exact solutions, extended the idea of arbitrary wall shear stress to the heat transfer problems. This work was pioneered by Fetecau *et al*. [14, 15], where they investigated free convection flow near a vertical plate that applies arbitrary shear stress to the fluid when the thermal radiation and porosity effects are taken into consideration. Soon after, Khan *et al.* [16, 17], extended this idea of arbitrary wall shear stress to the conjugate phenomenon of heat and mass transfer for the conducting viscous fluid near a vertical plate with ramped wall temperature and constant mass diffusion such that the fluid is passing through a porous medium. However, all these studies were performed in a non-rotating frame of reference.

Therefore, the main objective of this work is to provide exact solutions for the heat transfer analysis in MHD, an incompressible rotating viscous fluid over an infinite vertical plate that applies a time-dependent shear stress $f(t)$ to the fluid with ramped wall temperature. The solution corresponding to the general case $f(t)$, can be used to obtain solutions for many problems. Some special cases are extracted from the general solutions together with some particular solutions, if the angular velocity of the frame tends to zero, the solutions of some known problems are recovered. To illustrate the theoretical and practical importance of the studied problem, the effect of embedded parameters on the dimensionless velocity is graphically analyzed.

2. Governing equations and solutions

Let us consider an infinite vertical plate surrounded by an infinite mass of incompressible viscous fluid with ramped wall temperature. The $x-$axis of the coordinate system is taken along the plate and z axis is normal to the $x-$axis. Initially, $T_{1\infty}$ is the temperature for plate and the fluid. At time $t=0$, the plate applies a time dependent shear stress $f(t)$ to the fluid along the $x-$axis. Both the plate and fluid temperature aroused or lowered to $T_{1\infty}+(T_{1w}-T_{1\infty})\dfrac{t}{t_0}$ when $t<t_0$, and thereafter for $t\geq t_0$ remained at isothermal temperature T_{1w}. Meantime, the fluid together the plate, starts to rotate about y_1-axis with a constant angular velocity Ω. Under the usual Boussinesq' approximation, the unsteady flow is governed by the following set of partial differential equations which are

$$\frac{\partial u_1(y_1,t)}{\partial t}-2\Omega \upsilon_1(y_1,t)=v\frac{\partial^2 u_1(y_1,t)}{\partial y_1^2}+g\beta_{T_1}(T_1(y_1,t)-T_{1\infty})-\frac{\sigma B_0^2}{\rho}u_1(y_1,t); \qquad y_1,t>0, \qquad (1)$$

$$\frac{\partial \upsilon_1(y_1,t)}{\partial t}+2\Omega u_1(y_1,t)=v_1\frac{\partial^2\upsilon_1(y_1,t)}{\partial y_1^2}-\frac{\sigma B_0^2}{\rho}\upsilon_1(y_1,t), \quad (2)$$

$$\rho C_p\frac{\partial T_1(y_1,t)}{\partial t}=k\frac{\partial^2 T_1(y_1,t)}{\partial y_1^2}, \quad (3)$$

Where $u_1(y_1,t)$ and $v_1(y_1,t)$ are the velocity components along the $x-axis$ and y_1-axis respectively, $\Omega.$ is oscillation frequency; g is the gravitational acceleration, β_{T_1} the coefficient of volume expansion, v shows the kinematic viscosity, ρ is the constant density of the fluid, C_p is the specific heat at constant pressure, k is the coefficient of thermal conductivity, σ is the electric conductivity of the fluid; B_0 shows applied magnetics field and $T_1(y_1,t)$ is the temperature of the fluid.

We assume that no slip appears between the plate and fluid, thus the corresponding initial and boundary conditions are

$$\begin{aligned}
&u_1(y_1,0)=0,\ v_1(y_1,0)=0,\ T_1(y_1,0)=T_{1\infty};\quad \forall y_1\geq 0,\\
&\frac{\partial u_1(0,t)}{\partial y_1}=\frac{f(t)}{\mu},v_1(0,t)=0,\quad t>0,\\
&T_1(0,t)=T_{1\infty}+(T_{1w}-T_{1\infty})\frac{t}{t_0};0<t<t_0,\ T_1(0,t)=T_{1w},\ t\geq t_0,\\
&u_1(\infty,t)=0,\ \ v_1(\infty,t)=0,\ T_1(\infty,t)=T_{1\infty};\ t>0,
\end{aligned} \quad (4)$$

Where

$$\Pr=\frac{\mu C_p}{k},\upsilon=\frac{\mu}{\rho},M=\frac{\sigma B_0^2 t_0}{\rho} \quad (5)$$

Pr is a Prandtl number.

By introducing the following dimensionless variables

$$\begin{aligned}
&u_1^*=u_1\sqrt{\frac{t_0}{v}},\ \upsilon_1^*=\upsilon_1\sqrt{\frac{t_0}{v}},\ T_1^*=\frac{T_1-T_{1\infty}}{T_{1w}-T_{1\infty}},\ y_1^*=\frac{y_1}{\sqrt{vt_0}},\\
&t^*=\frac{t}{t_0},\ \ f^*(t^*)=\frac{t_0}{\mu}f(t_0t^*),
\end{aligned} \quad (6)$$

Into Eqs. (1) and (2) and dropping out the star notations it yields

$$\frac{\partial u_1\left(y_1,t\right)}{\partial t}-2Ek\upsilon(y_1,t)=\frac{\partial^2 u_1\left(y_1,t\right)}{\partial {y_1}^2}+GrT(y_1,t)-Mu_1(y_1,t), \quad (7)$$

$$\frac{\partial \upsilon_1\left(y_1,t\right)}{\partial t}+2Ek\upsilon(y_1,t)=\frac{\partial^2 \upsilon_1\left(y_1,t\right)}{\partial {y_1}^2}-M\upsilon_1(y_1,t), \quad (8)$$

$$\Pr\frac{\partial T_1\left(y_1,t\right)}{\partial t}=\frac{\partial^2 T_1\left(y_1,t\right)}{\partial {y_1}^2}, \tag{9}$$

Where

$$Gr=\frac{g\beta_{T_1}\left(T_{1w}-T_{1\infty}\right)\upsilon}{{U_0}^3}, Ek=\frac{\Omega\nu}{t_0}, t_0=\frac{\upsilon}{{U_0}^2}, \tag{10}$$

are the Grashof number and the dimensionless ratio $\frac{\Omega\upsilon}{t_0}$ is define as the Ekman number and t_0 is the characteristics time respectively.

The corresponding dimensionless initial and boundary conditions are:

$$\begin{aligned}
&u_1\left(y_1,0\right)=0, v_1\left(y_1,0\right)=0, T_1\left(y_1,0\right)=0; y_1\geq 0\\
&\left.\frac{\partial u_1}{\partial y_1}\right|_{y_1=0}=f\left(t\right), v_1\left(0,t\right)=0, T_1\left(0,t\right)=t; 0<t<1, T_1\left(0,t\right)=t; t\geq 1\\
&T_1\left(\infty,t\right)=0, u_1\left(\infty,t\right)=0, v_1\left(\infty,t\right)=0
\end{aligned} \tag{11}$$

The dimensionless temperature and the surface heat transfer rate respectively are given by [16, 17]

$$f\left(y_1,t\right)=\left(\frac{\Pr {y_1}^2}{2}+t\right)erfc\left(\frac{\sqrt{\Pr}y_1}{2\sqrt{t}}\right)-\frac{\sqrt{\Pr t}}{\sqrt{\pi}}y_1\exp\left(\frac{-\Pr {y_1}^2}{4t}\right) \tag{12}$$

and

$$\left.\frac{\partial T\left(y_1,t\right)}{\partial y_1}\right|_{y_1=0}=\frac{2\sqrt{\Pr}}{\sqrt{\pi}}\left(\sqrt{t}-\sqrt{t-1}H\left(t-1\right)\right) \tag{13}$$

where erf *c* (:) is the complementary error function of Gauss.

In order to obtained the velocity field, we use the complex velocity field $F(y_1,t)=u_1(y_1,t)+i\upsilon(y_1,t)$ and taking $H+M=2iEk, F(y_1,t)$ is the solution of the problem

$$\frac{\partial F\left(y_1,t\right)}{\partial t}=\frac{\partial^2 F\left(y_1,t\right)}{\partial {y_1}^2}+GrT\left(y_1,t\right)-\left(H+M\right)F\left(y_1,t\right) \tag{14}$$

$$F\left(y_1,t\right)=0, F\left(\infty,t\right)=0, \left.\frac{\partial F\left(y_1,t\right)}{\partial y_1}\right|_{y_1=0}=f\left(t\right), \tag{15}$$

Applying the Laplace transform to Eq.(14) and bearing in the mind the corresponding initial conditions for $F\left(y_1,t\right)$ yields

$$q\bar{F}\left(y_1,q\right)=\frac{\partial^2\bar{F}\left(y_1,q\right)}{\partial y_1^2}+Gr\bar{T}\left(y_1,q\right)-(H+M)\bar{F}\left(y_1,q\right), \tag{16}$$

where $\bar{F}\left(y_1,q\right)$ is the Laplace transform of the function $F\left(y_1,t\right)$. The corresponding boundary conditions (15), become

$$\bar{F}\left(\infty,q\right)=0,\ \frac{\partial\bar{F}\left(y_1,q\right)}{\partial y_1}\bigg|_{y_1=0}=\bar{F}\left(q\right), \tag{17}$$

$\bar{F}(q)$ being the Laplace Transform of function $f(t)$.

The solution of Eq. (16) under the conditions (17) is given by

$$\bar{F}(y_1,q)=\bar{F}_1(y_1,q)+\bar{F}_2(y_1,q)+\bar{F}_3(y_1,q), \tag{18}$$

Where

$$\begin{aligned}
\bar{F}_1(y_1,q)&=-\frac{\bar{F}(q)}{\sqrt{q+H}}\exp\left(-y_1\sqrt{q+H+M}\right),\\
\bar{F}_2(y_1,q)&=\frac{a_1\sqrt{q}}{q^2(q-a_2)\sqrt{q+H+M}}e^{-y_1\sqrt{q+H+M}}-\frac{a_1\sqrt{q}e^{-q}}{q^2(q-a_2)\sqrt{q+H+M}}e^{-y_1\sqrt{q+H+M}},\\
\bar{F}_3(y_1,q)&=\frac{a_3e^{-q}}{q^2(q-a_2)}e^{-y_1\sqrt{q\Pr}}-\frac{a_3}{q^2(q-a_2)}e^{-y_1\sqrt{q\Pr}},
\end{aligned} \tag{19}$$

With

$$a_1=\frac{Gr\sqrt{\Pr}}{\Pr-1},\quad a_2=\frac{H+M}{\Pr-1},\quad a_3=\frac{Gr}{\Pr-1}$$

By using the following function

$$r_1(y_1,t)=L^{-1}\left\{\frac{e^{-y_1\sqrt{q+H+M}}}{\sqrt{q+H+M}}\right\}=\frac{1}{\sqrt{\pi t}}\exp\left(-\frac{y_1^2}{4t}\right)e^{-(2Eki-M)t}=R_1(y_1,t)+iR_2(y_1,t), \tag{20}$$

With

$$R_1(y_1,t)=\frac{1}{\sqrt{\pi t}}\exp\left(-\frac{y_1^2}{4t}\right)\cos\left((2E_k-M)t\right),R_2(y_1,t)=-\frac{1}{\sqrt{\pi t}}\exp\left(-\frac{y_1^2}{4t}\right)\sin\left((2E_k-M)t\right), \tag{21}$$

and applying the convolution theorem, we have the inverse Laplace transform for the first term of Eq. (19), namely,

$$L^{-1}\{\bar{F}_1(y_1,q)\}=-f(t)*r_1(y_1,t)=S_1(y_1,t)+iS_2(y_1,t), \tag{22}$$

where $*$ represents convolution product and

$$\begin{aligned}
S_1(y_1,t)&=-\frac{1}{\sqrt{\pi}}\int_0^t\frac{f(t-s)e^{-\frac{y_1^2}{4s}}}{\sqrt{s}}\cos\left((2E_k-M)s\right)ds,\\
S_2(y_1,t)&=\frac{1}{\sqrt{\pi}}\int_0^t\frac{f(t-s)e^{-\frac{y_1^2}{4s}}}{\sqrt{s}}\sin\left((2E_k-M)s\right)ds.
\end{aligned} \tag{23}$$

For the second term from Eq. (19), we take $n=\left(2E_k-M\right)/\Pr-1,\ a_2=in$ and

$$r_2(y_1,q)=\frac{a_1\sqrt{q}}{q^2(q-a_2)}. \tag{24}$$

The inverse Laplace transform of Eq. (24) is:

$$r_2(y_1,t)=\frac{2a_1\sqrt{t}}{\pi}*e^{\text{int}}=\frac{2a_1\sqrt{t}}{\pi}*(\cos nt+i\sin nt)=S_3(y_1,t)+iS_4(y_1,t), \tag{25}$$

with

$$\begin{aligned}S_3(y_1,t)&=\frac{2a_1}{\pi}\int_0^t\sqrt{s}\cos\big(n(t-s)\big)ds=\frac{4a_1}{\pi}\int_0^{\sqrt{t}}x^2\cos\big(n(t-x^2)\big)dx,\\ S_4(y_1,t)&=\frac{2a_1}{\pi}\int_0^t\sqrt{s}\sin\big(n(t-s)\big)ds=\frac{4a_1}{\pi}\int_0^{\sqrt{t}}x^2\sin\big(n(t-x^2)\big)dx,\end{aligned} \tag{26}$$

we get

$$\begin{aligned}L^{-1}\{\bar{F}_2(y_1,q)\}&=\big(S_1(y_1,t)+iS_2(y_1,t)\big)*\big(S_3(y_1,t)+iS_4(y_1,t)\big)\\ &=(S_1*S_3-S_2*S_4)+i(S_1*S_4+S_2*S_3)\\ &=R_3(y_1,t)+iR_4(y_1,t),\end{aligned} \tag{27}$$

where

$$\begin{aligned}R_3(y_1,t)=&\left[\frac{4a_1}{\pi\sqrt{\pi}}\int_0^t\left[\frac{1}{\sqrt{s}}\exp\left(-\frac{y_1^2}{4s}\right)\cos(2Eks)\int_0^{\sqrt{t-s}}x^2\cos\big(n(t-s-x^2)\big)dx\right]ds\right]H(t)\\ &-\left[\frac{4a_1}{\pi\sqrt{\pi}}\int_0^{t-1}\left[\begin{array}{c}\frac{1}{\sqrt{s}}\exp\left(-\frac{y_1^2}{4s}\right)\\ \times\cos(2Eks)\int_0^{\sqrt{t-1-s}}x^2\cos\big(n(t-1-s-x^2)\big)dx\end{array}\right]ds\right]H(t-1)\\ &-\left[\frac{4a_1}{\pi\sqrt{\pi}}\int_0^{t-1}\left[\begin{array}{c}\frac{1}{\sqrt{s}}\exp\left(-\frac{y_1^2}{4s}\right)\\ \times\sin(2Eks)\int_0^{\sqrt{t-1-s}}x^2\sin\big(n(t-1-s-x^2)\big)dx\end{array}\right]ds\right]H(t-1)\\ &+\left[\frac{4a_1}{\pi\sqrt{\pi}}\int_0^{t}\left[\begin{array}{c}\frac{1}{\sqrt{s}}\exp\left(-\frac{y_1^2}{4s}\right)\\ \times\sin(2Eks)\int_0^{\sqrt{t-s}}x^2\sin\big(n(t-s-x^2)\big)dx\end{array}\right]ds\right]H(t),\end{aligned} \tag{28}$$

$$
\begin{aligned}
R_4(y_1,t) &= \left[\frac{4a_1}{\pi\sqrt{\pi}}\int_0^t\left[\frac{1}{\sqrt{s}}\exp\left(-\frac{y_1^2}{4s}\right)\cos(2Eks)\int_0^{\sqrt{t-s}}x^2\sin\left(n\left(t-s-x^2\right)\right)dx\right]ds\right]H(t) \\
&-\left[\frac{4a_1}{\pi\sqrt{\pi}}\int_0^{t-1}\left[\begin{array}{c}\frac{1}{\sqrt{s}}\exp\left(-\frac{y_1^2}{4s}\right)\\ \times\cos(2Eks)\int_0^{\sqrt{t-1-s}}x^2\sin\left(n\left(t-1-s-x^2\right)\right)dx\end{array}\right]ds\right]H(t-1) \\
&+\left[\frac{4a_1}{\pi\sqrt{\pi}}\int_0^{t-1}\left[\begin{array}{c}\frac{1}{\sqrt{s}}\exp\left(-\frac{y_1^2}{4s}\right)\\ \times\sin(2Eks)\int_0^{\sqrt{t-1-s}}x^2\cos\left(n\left(t-1-s-x^2\right)\right)dx\end{array}\right]ds\right]H(t-1) \\
&-\left[\frac{4a_1}{\pi\sqrt{\pi}}\int_0^{t}\left[\begin{array}{c}\frac{1}{\sqrt{s}}\exp\left(-\frac{y_1^2}{4s}\right)\\ \times\sin(2Eks)\int_0^{\sqrt{t-s}}x^2\cos\left(n\left(t-s-x^2\right)\right)dx\end{array}\right]ds\right]H(t).
\end{aligned}
\tag{29}
$$

The inverse Laplace of the third term, $\bar{F}_3(y_1,q)$ is obtained and given by:

$$
\begin{aligned}
L^{-1}\left\{\bar{F}_3(y_1,q)\right\} &= a_3\left(\left(t+\frac{\Pr y_1^2}{2}\right)\operatorname{erf}c\left(\frac{y_1\sqrt{\Pr}}{2\sqrt{t}}\right)-\frac{y_1\sqrt{\Pr}\sqrt{t}}{\sqrt{\pi}}\exp\left(\frac{-y_1^2\Pr}{4t}\right)\right)*e^{i\text{nt}} \\
&= a_3\left(\left(t+\frac{\Pr y_1^2}{2}\right)\operatorname{erf}c\left(\frac{y_1\sqrt{\Pr}}{2\sqrt{t}}\right)-\frac{y_1\sqrt{\Pr}\sqrt{t}}{\sqrt{\pi}}\exp\left(\frac{-y_1^2\Pr}{4t}\right)\right)\left(\cos nt+i\sin nt\right) \\
&= R_5(y_1,t)+iR_6(y_1,t),
\end{aligned}
\tag{30}
$$

where

$$
\begin{aligned}
R_5(y_1,t) &= \left[\begin{array}{c}a_3\int_0^t\cos\left(n(t-s)\right)\\ \left[\left(s+\frac{\Pr y_1^2}{2}\right)\operatorname{erf}c\left(\frac{y_1\sqrt{\Pr}}{2\sqrt{s}}\right)-\frac{y_1\sqrt{\Pr}\sqrt{s}}{\sqrt{\pi}}\exp\left(\frac{-y_1^2\Pr}{4s}\right)\right]ds\end{array}\right]H(t) \\
&-\left[\begin{array}{c}a_3\int_0^{t-1}\cos\left(n(t-1-s)\right)\\ \left[\left(s+\frac{\Pr y_1^2}{2}\right)\operatorname{erf}c\left(\frac{y_1\sqrt{\Pr}}{2\sqrt{s}}\right)-\frac{y_1\sqrt{\Pr}\sqrt{s}}{\sqrt{\pi}}\exp\left(\frac{-y_1^2\Pr}{4s}\right)\right]ds\end{array}\right]H(t-1),
\end{aligned}
\tag{31}
$$

$$R_6\left(y_1,t\right)=\begin{bmatrix} a_3\int_0^t \sin\left(n\left(t-s\right)\right) \\ \left[\left(s+\frac{\Pr y_1^2}{2}\right)\operatorname{erf}c\left(\frac{y_1\sqrt{\Pr}}{2\sqrt{s}}\right)-\frac{y_1\sqrt{\Pr}\sqrt{s}}{\sqrt{\pi}}\exp\left(\frac{-y_1^2\Pr}{4s}\right)\right]ds\end{bmatrix}H\left(t\right) \\ -\begin{bmatrix} a_3\int_0^{t-1} \sin\left(n\left(t-1-s\right)\right) \\ \left[\left(s+\frac{\Pr y_1^2}{2}\right)\operatorname{erf}c\left(\frac{y_1\sqrt{\Pr}}{2\sqrt{s}}\right)-\frac{y_1\sqrt{\Pr}\sqrt{s}}{\sqrt{\pi}}\exp\left(\frac{-y_1^2\Pr}{4s}\right)\right]ds\end{bmatrix}H\left(t-1\right). \tag{32}$$

Now, the velocity components $u_1\left(y_1,t\right)$ and $v_1\left(y_1,t\right)$ as the real and imaginary parts of the complex velocity field are obtained as

$$u_1\left(y_1,t\right)=\operatorname{Re}\left[F\left(y_1,t\right)\right]=S_1\left(y_1,t\right)+R_3\left(y_1,t\right)+R_5\left(y_1,t\right), \tag{33}$$

$$v_1\left(y_1,t\right)=\operatorname{Im}\left[F\left(y_1,t\right)\right]=S_2\left(y_1,t\right)+R_4\left(y_1,t\right)+R_6\left(y_1,t\right), \tag{34}$$

where $S_1\left(y_1,t\right),\ S_2\left(y_1,t\right),\ R_3\left(y_1,t\right),\ R_4\left(y_1,t\right),\ R_5\left(y_1,t\right) and\ R_6\left(y_1,t\right)$ are obtained in Eqs. (23), (28), (29), (31), and (32) respectively.

3. Plate with constant temperature

Equations (12), (33) and (34) give analytical expressions for the temperature and velocity of rotating fluid with ramped temperature. In order to highlight the effect of the ramped temperature distribution of the boundary on the flow, it is important to compare such a flow with the constant temperature. By adopting the same long but straight forward procedure, the temperature, rate of heat transfer and velocity are evaluated as:

$$T_1\left(y_1,t\right)=\operatorname{erf}c\left(\frac{y_1\sqrt{\Pr}}{2\sqrt{t}}\right), \tag{35}$$

$$\frac{\partial T_1\left(0,t\right)}{\partial y_1}=-\frac{\sqrt{\Pr}}{\sqrt{\pi t}}, \tag{36}$$

$$u_1\left(y_1,t\right)=\operatorname{Re}\left[F\left(y_1,t\right)\right]=-\frac{1}{\sqrt{\pi}}\int_0^t \frac{f\left(t-s\right)e^{-\frac{y_1^2}{4s}}}{\sqrt{s}}\cos\left(\left(2E_k-M\right)s\right)ds \\ +\frac{2a_1}{\pi}\int_0^t \frac{1}{\sqrt{s}}\exp\left(-\frac{y_1^2}{4s}\right)\begin{bmatrix}\cos\left(\left(2E_k-M\right)s\right)\int_0^{\sqrt{t-s}}\cos\left(n\left(t-s-x^2\right)\right)dx \\ +\sin\left(\left(2E_k-M\right)s\right)\int_0^{\sqrt{t-s}}\sin\left(n\left(t-s-x^2\right)\right)dx\end{bmatrix}ds \\ -a_3\int_0^t \cos\left(n\left(t-s\right)\right)\operatorname{erf}c\left(\frac{y_1\sqrt{\Pr}}{2\sqrt{s}}\right)ds, \tag{37}$$

$$
\begin{aligned}
v_1(y_1,t) = \operatorname{Im}\left[F(y_1,t)\right] &= \frac{1}{\sqrt{\pi}}\int_0^t \frac{f(t-s)e^{-\frac{y_1^2}{4s}}}{\sqrt{s}}\sin\left((2E_k-M)s\right)ds \\
&+\frac{2a_1}{\pi}\int_0^t \frac{1}{\sqrt{s}}\exp\left(-\frac{y_1^2}{4s}\right)\begin{bmatrix} \cos\left((2E_k-M)s\right)\int_0^{\sqrt{t-s}}\sin\left(n\left(t-s-x^2\right)\right)dx \\ -\sin\left((2E_k-M)s\right)\int_0^{\sqrt{t-s}}\cos\left(n\left(t-s-x^2\right)\right)dx \end{bmatrix}ds \\
&-a_3\int_0^t \sin\left(n(t-s)\right)\operatorname{erf}c\left(\frac{y_1\sqrt{\Pr}}{2\sqrt{s}}\right)ds.
\end{aligned} \tag{38}
$$

4. Special Cases

The solutions of velocity obtained in Section 3, are more general. Hence, in this section we intend to discuss some special cases of the present solutions together with some particular cases in order to gain more about the physical insight of the problem. So, we discuss the following important special cases whose technical relevance is well-known in the literature.

4.1. Case-I: $f(t) = f\Phi(t)$ (Constant Shear stress on the plate)

In this first case we take the arbitrary function $f(t) = f\Phi(t),$ where f is a dimensionless constant and $\Phi(\cdot)$ denotes the unit step function. $S_1(y_1,t)$ and $S_2(y_1,t)$ take the following forms

$$
\begin{aligned}
S_1(y_1,t) &= -\frac{f}{\sqrt{\pi}\sqrt{s}}\int_0^t \exp\left(-\frac{y_1^2}{4s}\right)\cos\left((2E_k-M)s\right)ds, \\
S_2(y_1,t) &= \frac{f}{\sqrt{\pi}\sqrt{s}}\int_0^t \exp\left(-\frac{y_1^2}{4s}\right)\sin\left((2E_k-M)s\right)ds.
\end{aligned} \tag{39}
$$

The velocity components $u_1(y_1,t)$ and $v_1(y_1,t)$ are given by Eqs. (33) and (34) in which the functions $S_1(y_1,t)$ and $S_2(y_1,t)$ are replaced by the expressions given in Eq. (39).

4,2. Particular Case $(\Omega = 0,)$, Non-rotating frame

In this case the Ekman number and megnatic term becomes zero, hence Eq. (39) reduces to

$$
S_1(y_1,t) = -\frac{f}{\sqrt{\pi}\sqrt{s}}\int_0^t \exp\left(-\frac{y_1^2}{4s}\right)ds, \tag{40}
$$

which is in good agreement with the earlier reported result by $[14, \text{Eq. } (23)]$.

4.3. Case-II: $f(t) = ft^a$ $(a > 0)$ (An accelerating Shear Stress)

In the final case, we take $f(t) = ft^a,$ in which the plate applies an accelerating shear stress to the fluid, $S_1(y_1,t)$ and $S_2(y_1,t)$ can be written as:

$$S_1(y_1,t) = -\frac{f}{\sqrt{\pi}\sqrt{s}}\int_0^t (t-s)^a \exp\left(-\frac{y_1^2}{4s}\right)\cos\left(\left(2E_k - M\right)s\right)ds,$$
$$S_2(y_1,t) = \frac{f}{\sqrt{\pi}\sqrt{s}}\int_0^t (t-s)^a \exp\left(-\frac{y_1^2}{4s}\right)\sin\left(\left(2E_k - M\right)s\right)ds. \quad (41)$$

4.4 **Particular Case** $(\Omega = 0,)$ (Non-rotating frame)

In this case the Ekman number becomes zero as in above case, hence Eq. (41) reduces to:

$$S_1(y_1,t) = -\frac{f}{\sqrt{\pi}\sqrt{s}}\int_0^t (t-s)^a \exp\left(-\frac{y_1^2}{4s}\right)ds, \quad (42)$$

which is equivalent to [15 , Eq. (33)].

5. Results and Discussion

In order to understand the physical aspects of the problem, the graphical results for both of the velocity components $u_1(y_1,t)$ and $v_1(y_1,t)$ are plotted for various parameters of interest such as Ekman number Ek, the Grashof number Gr and the Prandtl number $\Pr$. The graphs corresponding to the velocity components $u_1(y_1,t)$ and $v_1(y_1,t)$ are plotted in Figures 1 - 6, for constant shear stress on the plate. The primary and secondary velocity profiles at different values of Ekman number Ek for both ramped and constant wall temperature are shown in Figs. 1 and 2 , It is found that the velocity is decreasing with increasing values of Ek in both cases of ramped and isothermal plates. Physically, it is true due to the fact that increasing values of Ek causes the frictional force to increase which tends to resist the fluid flow and thus reducing its velocity. Figs. 3 and 4 illustrate the influence of Grashof number Gr on both type of velocity profiles. It is observed that velocity increases with increasing Gr. This implies that thermal buoyancy force tends to accelerate velocity for both ramped temperature and isothermal plates. Graphical results to show the influence of the Prandtl number $\Pr$ on the velocity profiles are presented in Figs. 5 and 6. It is observed that the velocity is a decreasing function with respect to $\Pr$ in both cases of ramped and constant wall temperature.

6. Conclusions

The purpose of this work is to study the heat transfer in Ekman boundary flow of a rotating, incompressible viscous fluid over an infinite plate with ramped wall temperature and applies an arbitrary shear stress to the fluid. Exact solutions of velocity (for both cases of ramped and constant wall temperature) are obtained using the Laplace transform technique and expressed in terms of exponential and complementary error functions. They satisfy all imposed initial and boundary conditions. These solutions are plotted in various figures for different parameters of interest. The following conclusions are extracted from this study.

- It is seen that velocity increases with increasing Gr.
- It is investigated that the velocity is a decreasing function with respect to $\Pr$ in both cases of ramped and constant wall temperature.
- It is found that the velocity is decreasing with increasing values of Ek in both cases of ramped and isothermal plates.

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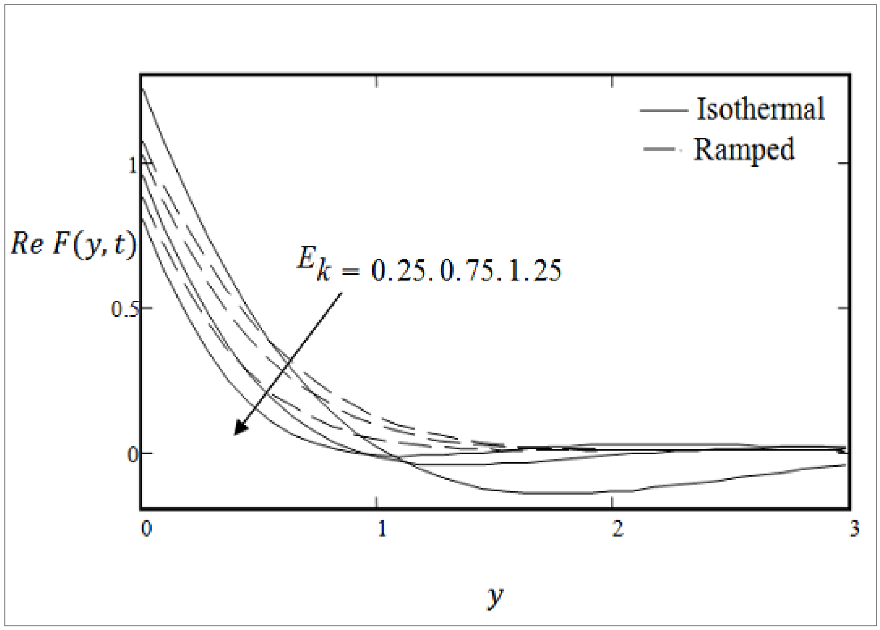

Fig. 1. Primary velocity profiles for different values of *Ek*

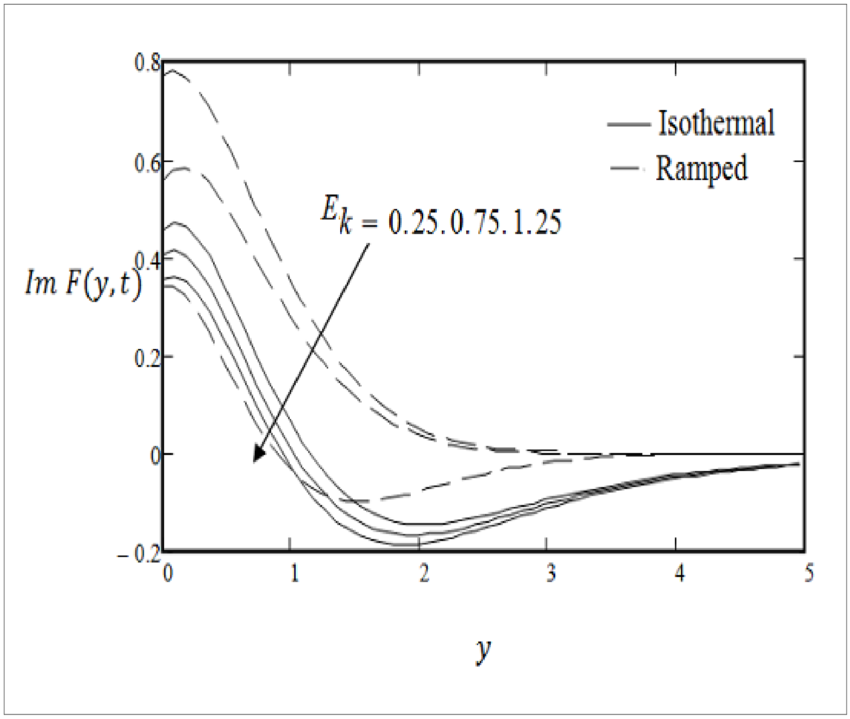

Fig. 2. Secondary velocity profiles for different values of *Ek* .

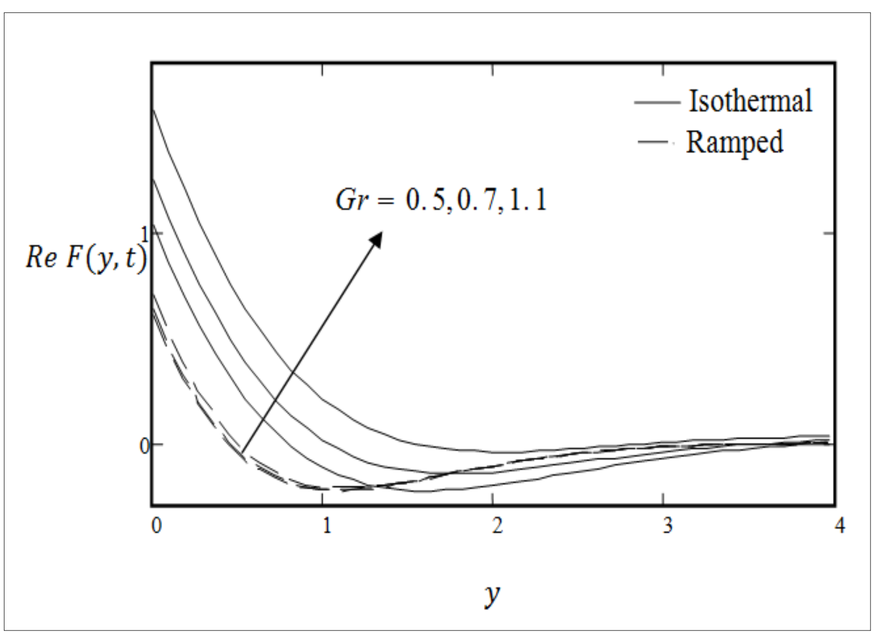

Fig. 3. Primary velocity profiles for different values of *Gr* .

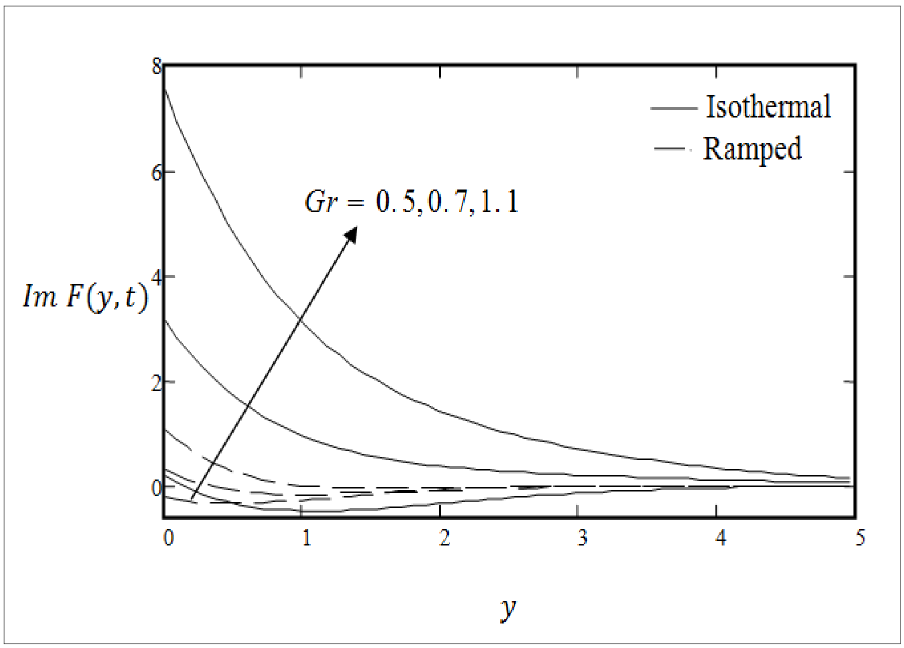

Fig. 4. Secondary velocity profiles for different values of Gr .

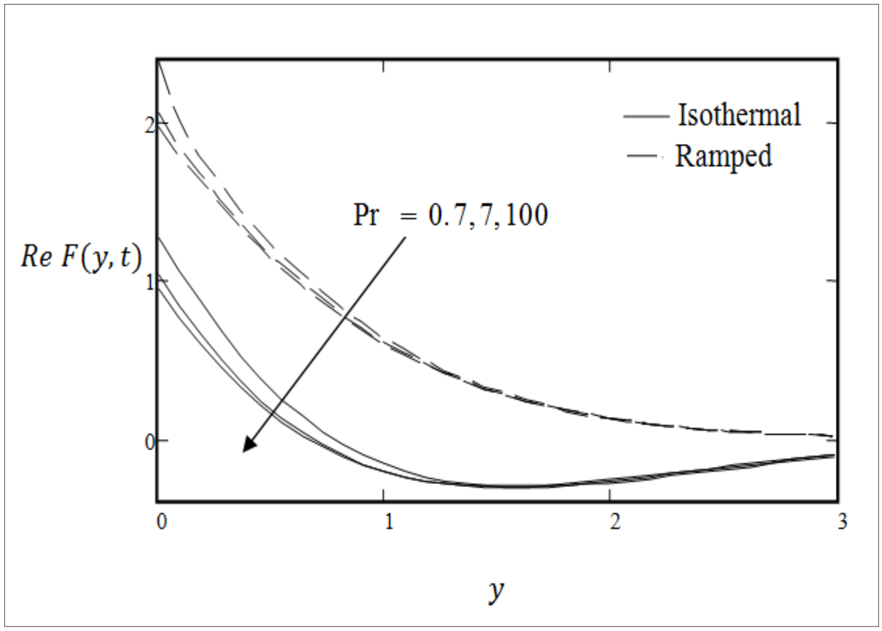

Fig. 5. Primary velocity profiles for different values of Pr .

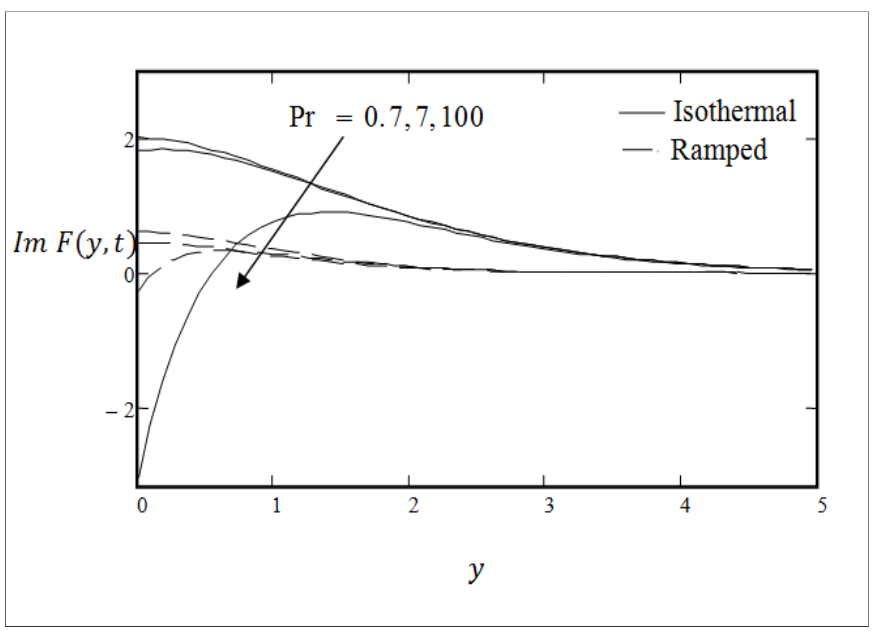

Fig. 6. Secondary velocity profiles for different values of Pr