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Stoke's First Problem for Casson Fluid between Two Side Walls over an Infinite Plate

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ABSTRACT

In this article, the unsteady flow of Casson fluid between two side walls normal to the plate is studied by using Fourier finite and infinite sine transform techniques. The bottom plate subjected to impulsive motion to the fluid at the time $t = 0^+$.Exact solutions have been established, which satisfy the governing equation and appropriate boundary and initial conditions. The obtained results are reduced to those solutions consequent to the flow over an infinite plate, when $h \to \infty$,and Newtonian fluid by making Casson parameter $\beta \to \infty$. Keeping in view the measure velocity value and shear stress in the middle of the channel, being not to be affected due to side walls, the required time to attain the steady-state and the distance between the side walls in such conditions is calculated graphically.

1 Introduction

The fluid that obeys Newton's law of viscosity and is easily described by the Navier Stock's equation is called Newtonian fluid. The flow of viscous fluid over an infinite plate with different initial and boundary conditions is discussed by numerous authors [1 - 4]. Puri et al. [5] obtained the closed-form solutions for the unsteady flow of viscous fluid in the presence of a magnetic field in a rotating frame by using the Laplace transform method. The starting solution of a three-dimensional unsteady MHD flow of viscous fluid in a rotating frame passing through a porous medium is obtained by Sulochana [6]. These flows also extended for non-Newtonian fluids by many authors [7 - 12]. In the mentioned references [1 - 12] the authors extended these flows for infinite plates. Moreover, the effect of side walls on the flow of fluid is very important and therefore, some researchers are interested to know about the distance for which the measured velocity value and shear stress are unaffected by the side walls. Due to the above importance, the researchers [13 - 16] studied the flow of fluid over an infinite plate between two side walls using different boundary and initial conditions. In nature, some non-Newtonian fluids are exits that behave like elastic solids i.e. at small shear stress no flow occurs, Casson fluid is one of such fluid. This fluid is quite famous recently because of its distinct features. In 1959, the first model was introduced by Casson to estimate the properties of the flow of pigment-oil suspensions [17]. Casson fluid can be defined as a shear-thinning liquid that is assumed to have infinite viscosity at a zero rate of shear, and a yield stress below which no flow occurs and a zero viscosity at an infinite rate of shear [18]. So, if, the magnitude of shear stress of Casson fluid increases from the yield shear stress, then behaves like a rigid body, and such fluids are treated as purely viscous because of very high viscosity [19]. The examples of Casson fluids are jelly, tomato sauce, soap, and honey, etc. Later on, Casson fluid is studied by many researchers for different flow configurations and situations. Among them, a solution for the unsteady flow of Casson fluid passed over a semi-infinite vertical plate with thermal and hydrodynamic slip conditions is obtained by Rao et al. [20]. Hussanan et al. [21] investigate the boundary layer flow of Casson fluid passed an oscillating vertical plate with Newtonian heating. Mustafa et al. [22] consider the unsteady flow of Casson

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fluid past over a moving flat plate with a heat transfer effect. A closed-form solution for the boundary layer flow of Casson fluid over a permeable shrinking / stretching sheet without and with an extended magnetic field was obtained by Bathacharyya et al. [23-24]. Raju [25] has studied the effect of an induced magnetic field on the stagnation flow of Casson fluid. The pioneering work on the closed-form solution for free convection and electrically conducting the flow of Casson fluid over an oscillating vertical plate passing through a porous medium is studied by Khalid et al [26]. Makanda [27-28] has discussed the effect of radiation as well as the chemical reaction of Casson fluid flow. Many researchers [29 - 32] studied the Casson fluid and obtained the solutions by using exact analytical methods or numerical methods under different boundary conditions. After a thorough review of the related literature, there have been no investigations on the effect of side walls on Casson fluid when the velocity is given on the boundary.

2 Mathematical Analysis of the Problem

Let us consider the unsteady two-dimensional flow of Casson fluid between two parallel side walls normal to the infinite plate and initially both, fluid and plate are stationary. At times $t = 0^+$,the bottom plate is subjected to impulsive motion to the fluid. Due to the impulsive motion, the fluid is gradually moved as shown in Fig. 1. The velocity field for the above description can be expressed as:

$$\vec{\mathbf{V}} = u\,(y,\, z,\, t)\,\mathbf{i}, \tag{1}$$

where $\mathbf{i}$ represents unit vector along the x-axis of the Cartesian coordinate system x, y and z. The rheological equation for the incompressible flow of Casson fluid is given by [31]:

$$\tau_{ij} = \left\{ \begin{array}{ll} 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi}}\right) e_{ij}, & \pi_c < \pi \\ 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}}\right) e_{ij}, & \pi_c < \pi \end{array} \right\}, \tag{2}$$

here, π represents the product of the component of the rate of deformation with itself, μ_B stands for plastic dynamic viscosity, e_{ij} denotes $(ij)^{th}$components of the deformation rate, p_y is the yield stress of the fluid and π_c represents the critical value of this product based on the non-Newtonian model. For such a flow, the constraint of incompressibility is
automatically satisfied and the governing equation after using equations (1) and (2), is

$$\frac{\partial u\,(y,\, z,\, t)}{\partial t} = \nu \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial^2 u\,(y,\, z,\, t)}{\partial y^2} + \frac{\partial^2 u\,(y,\, z,\, t)}{\partial z^2}\right),\;\; y,\, t > 0,\;\; z \in [0,\, d]\,, \tag{3}$$

where$\nu = \frac{\mu_B}{\rho}$ denotes kinematic viscosity,$d$represents distance between the walls, $\beta = \mu_B\sqrt{2\pi_c}/p_y$ is the Casson parameter and the appropriate initial and boundary conditions are:

$$\left. \begin{array}{c} u(y,\, z,\, 0) = 0 \quad \text{for } y > 0,\;\; z \in [0,\, d], \\ u(y,\, 0,\, t) = u(y,\, d,\, t) = 0 \text{ for } y,\, t > 0, \\ u(0,\, z,\, t) = u_0 \quad \text{for } t > 0,\;\; z \in (0,\, d), \end{array} \right\}, \tag{4}$$

$$\frac{\partial u\,(y,\, z,\, t)}{\partial y},\;\; u\,(y,\, z,\, t) \to 0,\;\; \text{as } y \to \infty,\;\; \text{for } t > 0,\, z \in [0,\, d]\,. \tag{5}$$

3 Procedure for Solution of Problem

We consider the flow of Casson fluid between two parallel side walls over an infinite plate which is situated at the planes $z = 0$ and $z = d$, the flow is confined in $(x,\, z)-$plane and between two side walls placed in the planes $z = d$ and $z = 0$. At time $t = 0^+$, the plate applies an impulsive motion to the fluid.

$$u(0,\, z,\, t) = u_0 \text{ for } t > 0,\;\; z \in (0,\, d). \tag{6}$$

Both sides of governing Eq. (3) is multiplying by$\sqrt{\frac{2}{\pi}}\cos\,(y\zeta)\,\sin\,(\lambda_n t)\,,$ where $\lambda_n = \frac{n\pi}{d}$, and then integrating the obtained result with respect to z and y from 0 to d and 0 to ∞ respectively, and keeping in mind the boundary and initial conditions Eq. (4) and (5), we get the following differential equation.

$$\frac{\partial u_{sn}(y,\, z,\, t)}{\partial t} + \nu\left(1 + \frac{1}{\beta}\right)\left(\zeta^2 + \lambda_n^2\right)\, u_{sn}\,(\zeta,\, t) = -\sqrt{\frac{2}{\pi}}\nu\zeta u_0 \left(1 + \frac{1}{\beta}\right) \frac{(-1)^n - 1}{\lambda_n},\;\; n = 1,\, 2,..,\tag{7}$$

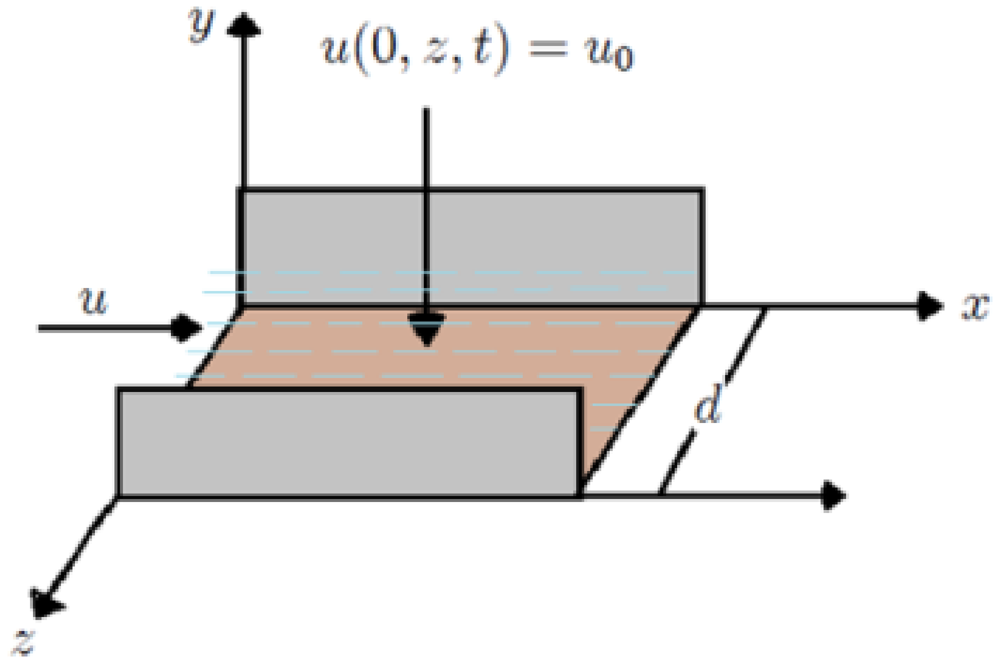

Figure 1: Schematic Diagram

here the Fourier finite and infinite sine transforms of $u\,(y,\, z,\, t)$is:

$$u_{sn}\,(\zeta,\, t) = \sqrt{\frac{2}{\pi}} \int\limits_0^d \int\limits_0^\infty u(y,\, z,\, t) \sin\left(\lambda_n z\right)\, \sin\left(y\zeta\right)\, dydz,\;\; n = 1,\, 2,\, 3..,\tag{8}$$

and it satisfies the following initial condition:

$$u_{cn}\,(\zeta,\, 0) = 0 \text{ for } \zeta > 0 \text{ and } n = 1,\, 2,\, 3....,\tag{9}$$

Eq. (7) can be expressed an ordinary differential equation in variable t for each fixed ξ, and by using the initial condition Eq. (9) , we get the solution of Eq. (7) as:

$$u_{sn}\,(\zeta,\, t) = \frac{-\zeta\sqrt{\frac{2}{\pi}}u_0}{\zeta^2 + \lambda_n^2} \frac{(-1)^n - 1}{\lambda_n} \left[1 - \exp\left(-\nu\left(1 + \frac{1}{\beta}\right)\,\left(\zeta^2 + \lambda_n^2\right)\right)\, t\right].\tag{10}$$

Now taking inverse Fourier finite and infinite sine transforms of Eq. (10), we get $u_s(y,\, z,\, t)$ under the form:

$$u_s(y,\, z,\, t) = \frac{-8u_0}{\pi d}\sum_{n=1}^{\infty}\frac{\sin(\lambda_n z)}{\lambda_n} \int_0^\infty \frac{\zeta \sin(\zeta y)}{\zeta^2 + \lambda_n^2} e^{-\nu\left(1+\frac{1}{\beta}\right)\,\left(\zeta^2+\lambda_n^2\right)\, t} d\zeta\; + \frac{8u_0}{\pi d}\sum_{n=1}^{\infty}\frac{\sin(\lambda_n z)}{\lambda_n} \int_0^\infty \frac{\zeta \sin(\zeta y)}{\zeta^2 + \lambda_n^2} d\zeta,\tag{11}$$

setting $d = 2h$, $m = 2n - 1$ and varying the origin of the coordinate system, substitute $z = z' + h$ and ignore the prime notation, Eq. (11) implies:

$$\begin{aligned} u_s(y,\, z,\, t) = \; & \tfrac{-4u_0}{\pi h} \sum_{n=1}^{\infty} \tfrac{(-1)^n \cos(\eta_m z)}{\eta_m} \int_0^\infty \tfrac{\zeta \sin(y\zeta)}{\zeta^2+\eta_m^2} d\xi + \tfrac{4u_0}{\pi h} \sum_{n=1}^{\infty} \tfrac{(-1)^n \cos(\eta_m z)}{\eta_m} \\ & \times \int_0^\infty \tfrac{\zeta \sin(y\zeta)}{\zeta^2+\eta_m^2} e^{-\nu\left(1+\frac{1}{\beta}\right)\,\left(\zeta^2+\eta_m^2\right)\, t} d\zeta, \end{aligned}\tag{12}$$

where $\eta_m = (2n - 1)\, \frac{\pi}{2h}$, now let $N = \frac{1}{1+\frac{1}{\beta}}$, and some identities used in this problem are (see also appendix A):

$$\int_0^\infty \frac{\zeta \sin\left(\zeta y\right)}{\zeta^2 + a^2} d\zeta = \frac{\pi}{2} e^{-ya}, \quad \text{Re}\,(a) \geq 0,\tag{13}$$

$$\int_0^\infty \frac{\zeta \sin(y\zeta)}{\zeta^2 + a^2} e^{-a\zeta^2} d\zeta = \frac{\pi}{2} e^{-ay} - \frac{\pi}{4} \left[\begin{array}{c} e^{ay} erfc\left(\frac{y}{2\sqrt{\nu t}} + a\sqrt{\nu t}\right) \\ +e^{-ay} erfc\left(\frac{y}{2\sqrt{\nu t}} - a\sqrt{\nu t}\right) \end{array} \right], \tag{14}$$

where $a^2 = \eta_m^2$ and $\nu = \nu\left(1 + \frac{1}{\beta}\right)$, by using the above identities we get the below expression for velocity field:

$$u_s(y, z, t) = \frac{u_0}{h} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cos(\eta_m z)}{\eta_m} \left[\begin{array}{c} e^{\eta_m y} erfc\left(\frac{y}{2\sqrt{\nu\left(1+\frac{1}{\beta}\right)t}} + \eta_m \sqrt{\nu\left(1 + \frac{1}{\beta}\right) t} \right) \\ +e^{\eta_m y} erfc\left(\frac{y}{2\sqrt{\nu\left(1+\frac{1}{\beta}\right)t}} - \eta_m \sqrt{\nu\left(1 + \frac{1}{\beta}\right) t} \right) \end{array} \right]. \tag{15}$$

Eq. (15) is the starting solution which satisfy all appropriate initial and boundary conditions.

4 Special Cases

4.1 Flow over an Infinite Plate

In the absence of the side walls, i.e. $h \to \infty$, into Eq. (15), we get the simplified form:

$$u_C(y, z, t) = u_0 erfc\left(\frac{y}{2\sqrt{\nu\left(1 + \frac{1}{\beta}\right) t}} \right). \tag{16}$$

4.2 Flow of Newtonian Fluid over an Infinite Plate

By putting $\beta \to \infty$ into Eq. (16), we get the velocity $u_s(y, z, t)$ for Stoke's 1st problem of Newtonian fluid:

$$u_N(y, z, t) = u_0 erfc\left(\frac{y}{2\sqrt{\nu t}}\right). \tag{17}$$

5 Numerical Results and Discussion

This paper describes about the study of the unidirectional and two-dimensional flow of Casson fluid between two parallel side walls perpendicular to an infinite bottom plate. The bottom plate provides an impulsive motion to the fluid. Due to this impulsive motion, the fluid moves gradually in its own plane. Exact solutions of velocity are determined for the described motion by using the integral transforms, namely Fourier finite and infinite sine transforms which satisfy all imposed initial and boundary conditions. Expressions for the flow of Casson fluid over an infinite plate by taking $h \to \infty$, are determined as a special case and also it has been noticed that for extremely large values of β i.e. $\beta \to \infty$, the non-Newtonian Casson fluid takes the behavior of the Newtonian fluid. In Fig. 2 we note that time is the increasing function of velocity i.e. when the time increases the velocity of the fluid flow also increases. Fig. 3 illustrates that the velocity of the fluid increases by increasing the distance between the side walls. The influence of the Casson fluid parameter on the velocity profile is shown in Fig. 4 and it is found that there is inverse variation between Casson parameter β and velocity. To find out the distance between the side walls for which the calculated value of velocity in the center of the channel is not affected by the existence of the side walls is shown in Fig. 5. The variation of time and Casson parameter β can be observed in Figs. 6 and 7 and we note from these figures that velocity is the decreasing function of Casson parameter and increasing function of time for the flow of fluid over an infinite plate. The comparison of Newtonian fluid with Casson fluid is shown in Fig. 8 and we note that by an extremely large value of β, i.e. $\beta \to \infty$, the graph of Casson fluid coincides with the graph of a Newtonian fluid. The units of material constants in Figs. 2 to 8 are SI units.

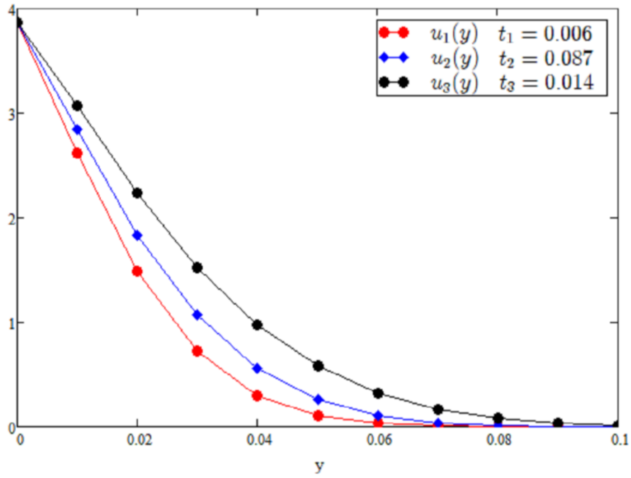

Figure 2: Eq. (15) presents the profile of velocities $u_1(y)$, $u_2(y)$, and $u_3(y)$ for various values of t and $u_0 = 4$, $\nu = 1.2$,$h = 0.1$, $\beta = 0.05$,$z = 0$

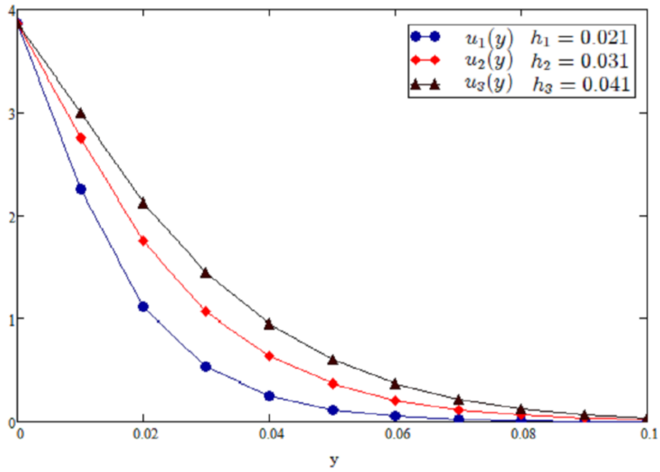

Figure 3: Eq. (15) presents the velocity profiles of $u_1(y)$, $u_2(y)$ and $u_3(y)$ for various values of h and $u_0 = 4$, $\nu = 1.2$, $t = 0.22s$, $\beta = 0.05$, $z = 0$

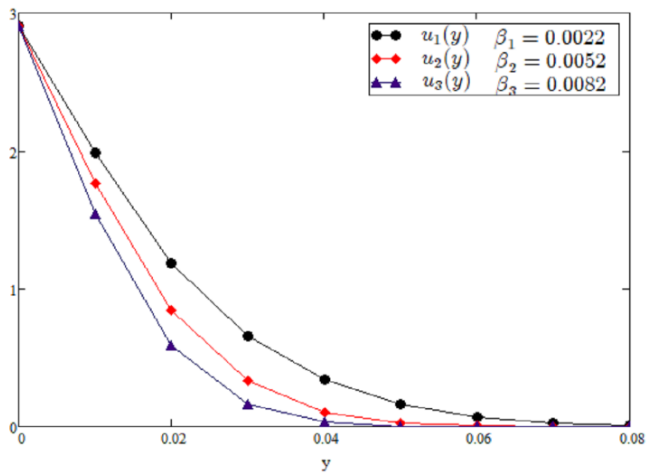

Figure 4: Eq. (15) presents the profiles of velocity $u_1(y)$, $u_2(y)$ and $u_3(y)$ for various values of β and $u_0 = 3$,$\nu = 0.002$,$t = 0.005s$,$h = 0.03$,$z = 0$

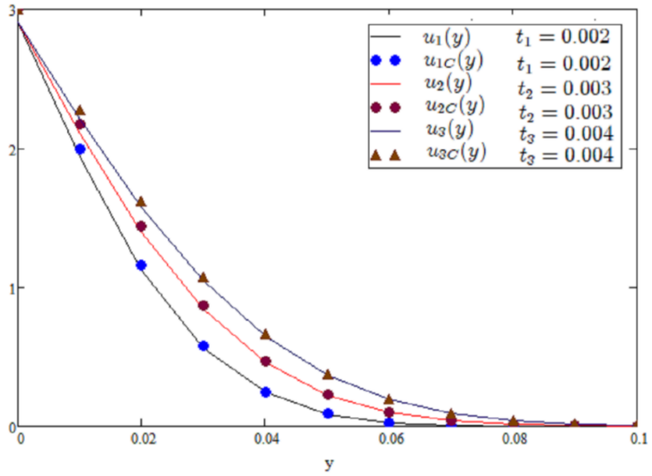

Figure 5: Eq. (15) presents the profiles of velocity $u_s(y, 0, t)$, $-$Curves $u_1(y)$, $u_2(y)$, $u_3(y)$ and Eq. (16) present the graph of $u_C(y, t)$ $-$ curves $u_{1C}(y)$, $u_{2C}(y)$ and $u_{3C}(y)$

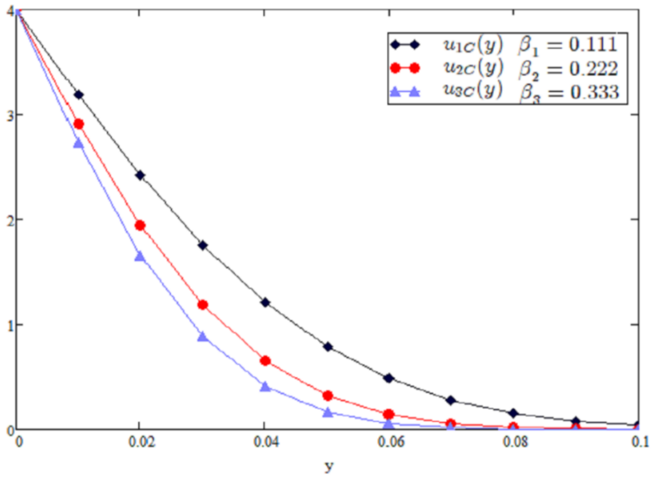

Figure 6: Eq. (16) presents the profile of velocities $u_C(y, 0, t)$ $-$ Curves $u_{1C}(y)$, $u_{2C}(y)$ and $u_{3C}(y)$ for $u_0 = 4$, $\nu = 2.5$, $t = 0.003$ and $z = 0$

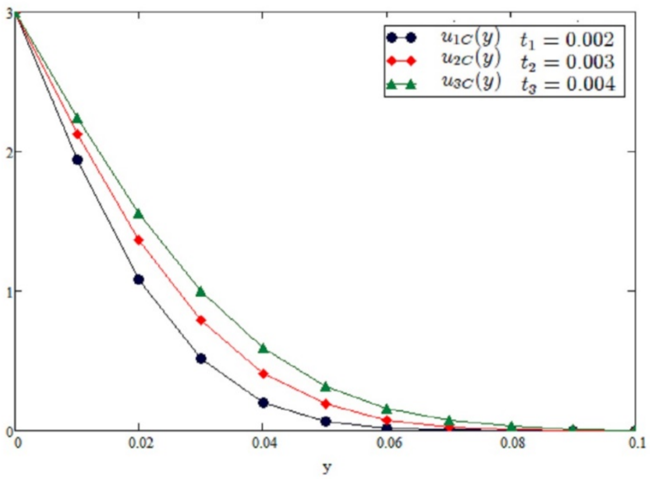

Figure 7: Eq. (16) presents the profile of velocities $u_C(y, 0, t)-$ Curves $u_{1C}(y)$, $u_{2C}(y)$ and $u_{3C}(y)$ for $u_0 = 4$, $\nu = 1.5$, $\beta = 0.111$ and $z = 0$

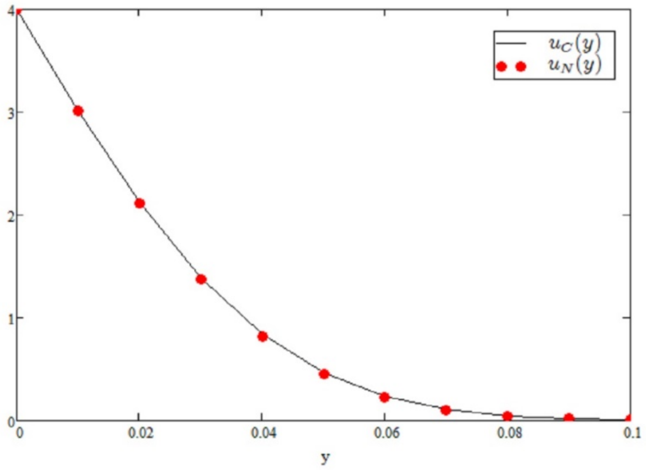

Figure 8: The comparison of velocities $u_C(y)$ and $u_N(y)$, given by Eqs. (16) and (17) for $u_0 = 4$, $t = 0.002$, $\nu = 2.5$, $\beta = 50$ and $z = 0$

6 Conclusion

Unsteady flow of a Casson fluid between two side walls of an infinite plate is examined. The plate provides an impulsive motion to the fluid. The plate applies an impulsive motion to the fluid. The general form of the exact solutions are determined with the help of integral transforms. The obtained solution satisfy all the imposed initial and boundary conditions. The Newtonian solutions are determined as limiting cases of the general solutions. Furthermore, they can also be used to give the solutions owing to the flow of fluid over an infinite plate that exerts the same impulsive motion to the fluid and various known solutions from the literature are obtained as limiting cases of our solutions. The results are plotted and it is found that velocity is the decreasing function of the Casson parameter.
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Conflicts of Interest The author declares no conflict of interest.

References

[1] M. E. Erdogan, "A note on an unsteady flow of a viscous fluid due to an oscillating plane wall," Int. J. Non. Linear. Mech., vol. 35, no. 1, pp. 1–6, 2000.

[2] Y. Zeng and S. Weinbaum, "Stokes problems for moving half-planes," J. Fluid Mech., vol. 287, pp. 59–74, 1995.

[3] C. Fetecau, D. Vieru, and C. Fetecau, "A note on the second problem of Stokes for Newtonian fluids," Int. J. Non. Linear. Mech., vol. 43, no. 5, pp. 451–457, 2008.

[4] M. E. Erdoğan and C. E. İmrak, "On the comparison of the solutions obtained by using two different transform methods for the second problem of Stokes for Newtonian fluids," Int. J. Non. Linear. Mech., vol. 44, no. 1, pp. 27–30, 2009.

[5] P. Puri and P. K. Kulshrestha, "Unsteady hydromagnetic boundary layer in a rotating medium," 1976.

[6] P. Sulochana, "Hall effects on unsteady MHD three dimensional flow through a porous medium in a rotating parallel plate channel with effect of inclined magnetic field," Am. J. Comput. Math., vol. 4, no. 05, pp. 396–405, 2014.

[7] K. R. Rajagopal, "A note on unsteady unidirectional flows of a non-Newtonian fluid," Int. J. Non. Linear. Mech., vol. 17, no. 5–6, pp. 369–373, 1982.

[8] M. E. Erdoğan, "Plane surface suddenly set in motion in a non-Newtonian fluid," Acta Mech., vol. 108, pp. 179–187, 1995.

[9] M. E. Erdogan, "On unsteady motions of a second-order fluid over a plane wall," Int. J. Non. Linear. Mech., vol. 38, no. 7, pp. 1045–1051, 2003.

[10] R. Bandelli, K. R. Rajagopal, and G. P. Galdi, "On some unsteady motions of fluids of second grade," Arch. Mech., vol. 47, no. 4, pp. 661–676, 1995.

[11] C. Fetecău and J. Zierep, "On a class of exact solutions of the equations of motion of a second grade fluid," Acta Mech., vol. 150, no. 1–2, pp. 135–138, 2001.

[12] C. Fetecău and C. Fetecău, "The Rayleigh–Stokes problem for heated second grade fluids," Int. J. Non. Linear. Mech., vol. 37, no. 6, pp. 1011–1015, 2002.

[13] M. E. Erdogan, "Effects of the side walls in generalized Couette flow," Appl. Mech. Eng., vol. 3, no. 2, pp. 271–286, 1998.

[14] M. E. Erdogan and C. E. Lmrak, "Effect of the side walls on the unsteady flow in A duct of rectangular cross-section," Math. Comput. Appl., vol. 9, no. 3, pp. 443–448, 2004.

[15] M. E. Erdoğan and C. E. İmrak, "Effects of the side walls on starting flows in ducts," Int. J. Non. Linear. Mech., vol. 40, no. 1, pp. 103–111, 2005.

[16] M. E. Erdoğan and C. E. Imrak, "The effects of the side walls on the flow of a second grade fluid in ducts with suction and injection," Int. J. Non. Linear. Mech., vol. 42, no. 5, pp. 765–772, 2007.

[17] N. Casson, "Flow equation for pigment-oil suspensions of the printing ink-type," Rheol. disperse Syst., pp. 84–104, 1959.

[18] R. K. Dash, K. N. Mehta, and G. Jayaraman, "Casson fluid flow in a pipe filled with a homogeneous porous medium," Int. J. Eng. Sci., vol. 34, no. 10, pp. 1145–1156, 1996.

[19] K. Bhattacharyya, T. Hayat, and A. Alsaedi, "Exact solution for boundary layer flow of Casson fluid over a permeable stretching/shrinking sheet," ZAMM-Journal Appl. Math. Mech. für Angew. Math. und Mech., vol. 94, no. 6, pp. 522–528, 2014.

[20] A. Subba Rao, V. Ramachandra Prasad, N. Bhaskar Reddy, and O. Anwar Bég, "Heat transfer in a Casson rheological fluid from a semi-infinite vertical plate with partial slip," Heat Transf. Res., vol. 44, no. 3, pp. 272–291, 2015.

[21] A. Hussanan, M. Zuki Salleh, R. M. Tahar, and I. Khan, "Unsteady boundary layer flow and heat transfer of a Casson fluid past an oscillating vertical plate with Newtonian heating," PLoS One, vol. 9, no. 10, p. e108763, 2014.

[22] M. Mustafa, T. Hayat, I. Pop, and A. Aziz, "Unsteady boundary layer flow of a Casson fluid due to an impulsively started moving flat plate," Heat Transf. Res., vol. 40, no. 6, pp. 563–576, 2011.

[23] K. Bhattacharyya, "Boundary layer stagnation-point flow of Casson fluid and heat transfer towards a shrinking/stretching sheet," Front. Heat Mass Transf., vol. 4, no. 2, 2013.

[24] K. Bhattacharyya, T. Hayat, and A. Alsaedi, "Exact solution for boundary layer flow of Casson fluid over a permeable stretching/shrinking sheet," ZAMM-Journal Appl. Math. Mech. für Angew. Math. und Mech., vol. 94, no. 6, pp. 522–528, 2014.

[25] C. S. K. Raju, N. Sandeep, and S. Saleem, "Effects of induced magnetic field and homogeneous–heterogeneous reactions on stagnation flow of a Casson fluid," Eng. Sci. Technol. an Int. J., vol. 19, no. 2, pp. 875–887, 2016.

[26] A. Khalid, I. Khan, A. Khan, and S. Shafie, "Unsteady MHD free convection flow of Casson fluid past over an oscillating vertical plate embedded in a porous medium," Eng. Sci. Technol. an Int. J., vol. 18, no. 3, pp. 309–317, 2015.

[27] G. Makanda, S. Shaw, and P. Sibanda, "Effects of radiation on MHD free convection of a Casson fluid from a horizontal circular cylinder with partial slip in non-Darcy porous medium with viscous dissipation," Bound. Value Probl., vol. 2015, no. 1, pp. 1–14, 2015.

[28] G. Makanda, S. Shaw, and P. Sibanda, "Diffusion of chemically reactive species in Casson fluid flow over an unsteady stretching surface in porous medium in the presence of a magnetic field," Math. Probl. Eng., vol. 2015, 2015.

[29] F. Mabood, S. Shateyi, and W. A. Khan, "Effects of thermal radiation on Casson flow heat and mass transfer around a circular cylinder in porous medium," Eur. Phys. J. Plus, vol. 130, pp. 1–12, 2015.

[30] S. T. Mohyud-Din and S. I. Khan, "Nonlinear radiation effects on squeezing flow of a Casson fluid between parallel disks," Aerosp. Sci. Technol., vol. 48, pp. 186–192, 2016.

[31] S. Pramanik, "Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation," Ain shams Eng. J., vol. 5, no. 1, pp. 205–212, 2014.

[32] H. Khan, M. Qayyum, O. Khan, and M. Ali, "Unsteady squeezing flow of Casson fluid with magnetohydrodynamic effect and passing through porous medium," Math. Probl. Eng., vol. 2016, 2016.

Appendix A

$$\int_0^\infty \frac{\xi \sin(y\xi)}{\xi^2 + a^2} d\xi = \frac{\pi}{2} e^{-ay}$$

$$\int_0^\infty \frac{\xi \sin(y\xi)}{\xi^2 + a^2} e^{-\nu\left(\xi^2+a^2\right)t} d\xi = \frac{\pi}{2} e^{-ay} - \frac{\pi}{4} \left[\begin{array}{c} e^{ay} erfc\left(\frac{y}{2\sqrt{\nu t}} + a\sqrt{\nu t}\right) \\ +e^{-ay} erfc\left(\frac{y}{2\sqrt{\nu t}} - a\sqrt{\nu t}\right) \end{array} \right]$$

$$\frac{2}{h} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cos(\eta_m z)}{\eta_m} = 1$$

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