

Numerical Approach to Thermo-Convective Micro-Polar Fluid with Radiation in Permeable Medium

Muhammad Shuaib^{1,*}, Farman Ali Shah¹, Hijab ur Rehman¹

¹City University of Science and Information Technology, Peshawar, Pakistan

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ABSTRACT

A steady, incompressible, and thermo-convective flow of micro-polar fluid over a stretching permeable sheet with heat and mass transfer under effects of radiation, Soret, Schmidt, and Dufour numbers have been analyzed. The modeled governing equations, of the classical Navier-Stokes, are coupled with micro rotation, temperature, and concentration equations, in the form of Partial Differential Equations (PDE's), along with initial and boundary conditions, are transformed into a system of nonlinear coupled Ordinary Differential Equations (ODE's) by using an appropriate transformation. The numerical solution is obtained by using the Parametric Continuation Method (PCM). For the validity of the scheme, the results are compared with a numerical package bvp4c. It has been observed that both the results are in the best agreement with each other. The effects of associated parameters on the dimensionless velocity, micro-rotation, temperature, and concentration profiles are discussed and depicted graphically. It has been detected that the permeability parameter gives rise to the micro-rotation profile.

1 Introduction

The transfer of heat along the thin film flow of micro-polar fluid has a great impact on research in the field of electronics and especially the exchange of heat inside the circuits of electronic devices, due to uncountable applications described in [1-3]. To maximize and improve the allowance of heat transfer of patterns flow, extension in the surface flow has been highly effective. In industries like automobiles, fabrics, and heavy machinery, in all such areas heat transfer has great importance also in the designing of manufacturing equipment, jets, army emanations, spaceships, turbines of different power plants, and nuclear reactors are the phenomena of heat exchange. Examining the impacts of radiation on the boundary layer of fluids is not an easy job to deal. The phenomenon of heat transfer was explained by Cengel [4], in the encyclopaedia of energy engineering and technology. The behaviour of the transfer of heat and flow of the fluids on Sinusoidal-Corrugated channels are numerically investigated by Khoshvaght-Aliabadi [5]. The Micro-polar fluids were first introduced by Eringen [6], who explains the micro-rotation effects on the micro-structures because the theory presented by Navier and Stokes does not explain, precisely the properties associated with polymeric fluids, colloidal fluids, suspension, and solutions, liquids containing crystals and fluids with additives. Eringen [7] further explained the thermo-micropolar fluids, with the behaviour of micro-structures on the film flow of fluids. Stokes [8] presented a theory of Fluids with Micro-structures. Researchers are studying the effects of radiations on the boundary layer of fluids over plates, radiations on conducting micropolar fluid over uniform and stretched surfaces have been explained by A.Eldahab [9]. The radiations on stretching plates with varying viscosity have been studied

*Corresponding author: Muhammad Shuaib

email address: mshoaib01@gmail.com

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by A.Eldahab and E.Gendy [10]. Micropolar fluids with heat transfer through a porous medium in the presence of radiations were discussed by A.Eldahab [11]. The micropolar fluid flow upon consistently moving plate along with radiations was analyzed by Raptis.A [12]. The transfer of heat of a micro-polar fluid along with radiations was explained by Raptis. A et al. [13]. The flow model of a micropolar fluid over a permeable medium has vast applications, alloys, fabrics, untreated wood, rocks, porous polymers, and blends of polymers. The effects of viscous resistance on the boundary of the flow, with inertia force and heat transfer in a constant porosity, were examined by Reddy. Ramakrishna and Raju [14]. A. Raptis [15] studied boundary layer flow through an absorbent medium. A. Eldahab and E.Gendy [16] explain heat transfer through convection with a magnetic field. Ariman et al. [17]. Ahmadi [18] gives solutions of a micropolar fluid passing over a partially unbounded plate along with the effects of micro-inertia. Soundalgekar et al [19] explain the stream and exchange of temperature over a ceaselessly moving plate. Gorla [20] investigated, steady heat transfer in the micropolar fluid using similarity techniques. The convection flow of micro-polar fluid on a perpendicular plate was studied by Rees and Pop [21]. Kim [22] discussed the unsteady flow with free convection on perpendicular plates upon the absorbent medium. Singh [23] explained the same work using the finite difference method.

Nomenclature

Description	Symbols	Description	Symbols
The micro-rotation constant	G_1	Free stream velocity	U
concentration on surface	C_∞	Wall temperature field	T_w
Temperature field	T	Forchheimer inertia	C_r
Wall concentration field	C_w	Concentration field	C
Coupling constant	k_c	Fluid thermal diffusivity	k
Temperature on surface	T_∞	Stretching velocity	U_0
Thermal radiation parameter	R	Specific heat at constant pressure	c_p
concentration susceptibility	c_s	Thermal diffusion ratio	K_T
Concentration molecular diffusion	D_m	Fluid's mean temperature	T_m
Scaled boundary layer coordinate	η	Fluid density	ρ
Angular velocity of fluid particles	σ	uniform thickness of boundary layer	ϕ, δ
Kinematic viscosity	ν	Dynamic viscosity	μ
Porosity parameters	δ	-	-

We know that, the modeling of natural phenomena leads to a partial differential equation which is a highly non-linear system, it is obvious that solving linear problems by analytic approach is not an easy job. As there are perturbation techniques discussed and explained in [24, 25] used in different areas but still it is difficult to apply them to every non-linear problem because it depends on some parameters which are not present in every problem, so some methods which are other than perturbative techniques are small parameter method [26], the delta expansion method [27], Adomian decomposition method (ADM) [28], have been developed but it cannot give us the rate of convergence and region of solution in an easy way. Liao [29-32], in 1992 gives the idea of an analytic method namely the homotopy analysis method (HAM), for the solution of non-linear problems, HAM gives quick convergence and accuracy as compared to the above methods. Mitri Prashant G and Tawade [33] used several geometries to explain the flow. Khan et al. [34] also described the impacts of different variables on the flow of micro-polar fluid. Mahmood, Tahir, and Nargis [35] analyzed the impacts of different variables on the flow of several fluids in their thin film flow. Rashidi, MM, and Mohimanian Pour, presented an analytical solution by using the DTM-Pade technique of micro-polar fluid through permeable media with the effects of radiations [36]. Rashidi.MM et al. [37] also handled expository surmised solutions to the exchange of heat in the micropolar liquid inside a penetrable medium along radiation impacts. Turkyilmazoglu [38] discussed micropolar fluid flow upon the permeable shrinking sheet. Abdul Gaffar et al. [39] give an idea of convective flow and temperature with non-linearity. Ibrahim et al. [40] discussed the micropolar and nanofluids with slip conditions along with the Soret and Dufour effects. In our study, we supposed the heat exchange in a micro-polar fluid with the impacts of radiation in a penetrable medium. Our manuscript consists of coupled energy and concentration equations. The system has been transferred to a system of ordinary differential equations once, and then these have been solved by numerical techniques. For this purpose, the modelled equations are tackled numerically by using two different numerical techniques, the predictor-corrector method and the bvp4c method. The obtained conclusions are compared and discussed with the help of graphs, which

shows reasonable settlement with each other.

2 Mathematical Modelling

Let's consider the steady film flow of viscous and incompressible micro-polar fluid flow which is being stretched with a velocity $U_0=bx$, is a constant and x where the rate of stretching is denoted by $b > 0$, display the direction of the linear velocity by which the plate is being stretched. Let δ , be the thickness and is chosen to be uniform. The medium is permeable in the semi-infinite horizontal plate in the region $y > 0$ as shown in figure (1). The temperature $\tau = (\tau_w - \tau_\infty) + \tau_\infty$ and concentration $C = (C_w - C_\infty) + C_\infty$, varies apart from the surface of the plate. τ_w and C_w is the temperature and concentration of the plate, τ_∞ and C_∞ is the temperature and concentration of the surrounding respectively. Moreover, it is assumed that the flow is gripping and radiative. The thermal radiations are considered besides x-coordinate, and no radiations along y direction, the required equations for the flow of the problem are as follows:

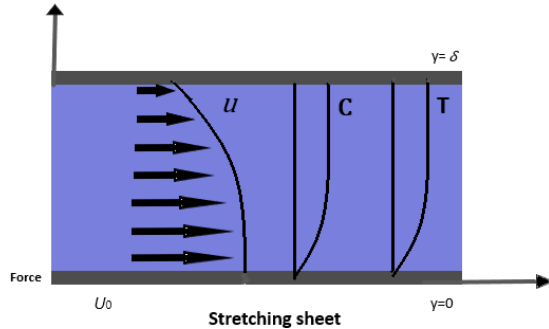


Figure 1: Geometry of the flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial^2 y} + k_c \frac{\partial \sigma}{\partial y} + \frac{\nu \phi}{K} (U - u) + C_r \phi (U^2 - u^2), \quad (2)$$

$$G_1 \frac{\partial^2 \sigma}{\partial^2 y} - 2\sigma - \frac{\partial u}{\partial y} = 0, \quad (3)$$

$$u \frac{\partial \tau}{\partial x} + v \frac{\partial \tau}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 \tau}{\partial^2 y} + \frac{16\sigma^*}{3\rho c_p k^*} \frac{\partial^2 \tau}{\partial^2 y} + \frac{\nu D_m k_T}{T_m c_s c_p} \frac{\partial^2 C}{\partial^2 y}, \quad (4)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial^2 y} + \frac{D_m k_T}{T_m} \frac{\partial^2 \tau}{\partial^2 y}, \quad (5)$$

boundary conditions for the two dimensional flow is given as:

$$\left. \begin{aligned} u=U_0, \quad v=0, \quad \sigma=0, \quad \tau=\tau_w, \quad C=C_w, \quad \text{at } y=0. \\ u_y=\sigma_y=\tau_y=C_y=0, \quad v=\delta_x \quad \text{at } y=\delta. \end{aligned} \right\}. \quad (6)$$

here the term associated with thermal radiation is defined as:

$$q_r = \frac{4\sigma^*}{3K^*} \frac{\partial \tau^4}{\partial y}, \quad (7)$$

where K^* and σ^* is the coefficient of mean absorption and Stefan Boltzmann constant respectively. Ignoring the second and higher terms in Taylor's series we supposed only the term τ_4 in Taylor's series about τ_1 , which represents temperature of free surface, we get:

$$\tau^4 \simeq 4\tau_1^3 \tau^* - 3\tau_1^4, \quad (8)$$

by using Eqs. (7) and (8), Eq. (4) becomes,

$$u \frac{\partial \tau}{\partial x} + v \frac{\partial \tau}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 \tau}{\partial^2 y} + \frac{16\sigma^* \tau_1^3}{3\rho c_p k^*} \frac{\partial^2 \tau}{\partial^2 y} + \frac{\nu D_m k_T}{T_m c_s c_p} \frac{\partial^2 C}{\partial^2 y}. \quad (9)$$

The corresponding similarity transformations are:

$$\psi(x, y) = (2\nu U_0 x)^{\frac{1}{2}} f(\eta), \quad u = \psi_y, \quad v = -\psi_x, \quad \sigma = \left(\frac{U_0}{2\nu x}\right)^{\frac{1}{2}} U_0 g(\eta) \quad \text{and} \quad \eta = \left(\frac{U_0}{2\nu x}\right)^{\frac{1}{2}} y. \quad (10)$$

Also the temperature and concentration for the thin film flow are

$$\theta(\eta) = \left(\frac{\tau - \tau_\infty}{\tau_w - \tau_\infty} \right) \quad \text{and} \quad \varphi(\eta) = \left(\frac{C - C_\infty}{C_w - C_\infty} \right). \quad (11)$$

Putting Eq.(10) and Eq.(11) into Eqs.(1)-(6). The following system of ordinary differential equations will be obtained, by using similarity transformation:

$$f''' + \Delta g' + ff'' + \frac{1}{M}(1 - f') + N(1 - f'^2) = 0, \quad (12)$$

$$Gr g'' - 2(2g + f'') = 0, \quad (13)$$

$$(3R + 4)\theta'' + 3RPr(Du\phi'' + f\theta') = 0, \quad (14)$$

$$\phi'' + Sr\theta'' - Scf\phi' = 0. \quad (15)$$

The changed boundary conditions are as follows:

$$\left. \begin{aligned} f(0) = 1, \quad g(0) = 1, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1 \quad \text{at} \quad y = 0 \\ f''(1) = f(1) = g'(1) = \theta'(1) = \phi'(1) = 0 \quad \text{at} \quad y = 1. \end{aligned} \right\}. \quad (16)$$

Where f and g are the dimensionless velocity and micro rotation angular velocity functions, θ and φ are temperature and concentration functions, $\Delta = \frac{k_1}{\nu}$ is the vortex viscosity parameter, $Mr = \frac{Ka}{2\phi\nu}$ is the permeability parameter, $Nr = \frac{2\phi C_r u_0}{a}$ is the inertia coefficient parameter, $Gr = \frac{G_1 a}{\nu}$ denotes micro rotation parameter, $Pr = \frac{\rho\nu c_p}{k}$ denotes Prandtl number, $R = \frac{4\sigma^* T_1^3}{K^*}$ denotes radiations parameter, $Sc = \frac{\nu}{D_m}$ denotes schmidt number, $Sr = \frac{D_m K_T (T_w - T_0)}{\nu T_m (C_w - C_0)}$ denotes soret number and $Df = \frac{D_m K_T (C_w - C_0)}{\nu T_m (T_w - T_0)}$ denotes dufour number. The quantities, local skin friction coefficient, the local Nusselt number, which is the non dimensional rate of heat transfer and the mass transfer rate is called local Sherwood number, these quantities have physical interpretation which are defined as:

$$C_f = \frac{\tau_w^s}{\rho u_w^2}, \quad Nu_x = \frac{xq_w}{K(\tau_w - \tau_\infty)}, \quad Sh_x = \frac{xq_m}{D_m(C_w - C_\infty)}. \quad (17)$$

where τ_w^s , q_w and q_m are the shear stress along the walls, the heat fluctuation and the mass fluctuation at the boundary, which is given by

$$\tau_w^s = \mu \left(\frac{\partial u}{\partial y} \right)^{y=0}, \quad q_w = - \left(K \frac{\partial \tau}{\partial y} \right)^{y=0}, \quad q_m = - \left(D_m \frac{\partial C}{\partial y} \right)^{y=0}. \quad (18)$$

With μ being the dynamic viscosity, then from Eqs.(7) and (18) into Eq.(17), we get

$$C_f \sqrt{Re_x} = -f''(0), \quad \frac{Nu_x}{\sqrt{Re_x}} = -\theta'(0), \quad \frac{Sh_x}{\sqrt{Re_x}} = -\phi'(0). \quad (19)$$

Here, $Re = \frac{U_0 x}{\nu}$ is Reynold's number. the values of the above parameters of physical interest, for all the embedded parameters, are represented in Table 1.

Table 1: Values of C_f , Nu_x and Sh_x

Δ	Mr	Nr	R	Pr	Sc	Sr	D_u
0.3	0.8	0.3	0.3	0.3	0.3	0.3	0.3
0.4	0.8	0.3	0.3	0.3	0.3	0.3	0.3

3 Results and Discussion

The thin film, stream of micro-polar fluid in permeable medium along the effects of coupled temperature and concentration fields, with extending lower plate is explored. The system of Equations (12-15) with

Table 2: Comparison of present result with Refs. [10,11,14] where $r=0.2$, $\phi=0$, $\rho_s=1$, $\rho_f=1$, $(C_p)_s=1$, $(C_p)_f=1$, $K_{nf}=1$, $K_f=1$, $A=\zeta=0$, $\xi=0$, $\epsilon=0$

Author	$f'(0)$	$-g'(0)$	$-h'(\infty)$	$-\theta(0)$
Present	0.5170	0.6221	0.8889	0.4003
Andersson [38]	0.510	0.616	0.883	-
Ming [39]	0.51021	0.61591	0.88230	0.39632
Xun et al. [17]	0.510231	0.615921	-	0.396271
Hayat et al. [40]	0.5109	0.61598	-	0.3959

corresponding boundary condition Equation (16) are tackled numerically, i.e. Predictor corrector technique and bvp4c technique, for comparison of substantial parameters, like Δ , R , Mr , Nr , Gr , Pr , Sc , Sr and D_u .

The physical parameters impeded in our model are explained. The numerical techniques presents best resemblance. The conduct of implanted parameters on capacities like velocity, temperature, micro-rotation field and concentration areas are watched and dissected through graphs, Figure (2-5). Figure (1) shows the geometrical layout of the problem. In figure 2(a) the act upon of Δ on dimensionless velocity $f(\eta)$ is represented, clearly Δ is inversely related with viscosity, for greater values of Δ the thickness decays, as a result the velocity of liquid increments. The impact of permeability Mr on the $f(\eta)$ is depicted in Figure 2(b). As we know that the greater values for Mr leads to highly porous medium, so it will obviously decelerates the fluid flow as a result reduction in the velocity occurs. The influence of inertia coefficient parameter Nr is shown in Figure 2(c). As Nr is directly proportional to the velocity, so clearly by expanding values of Nr the velocity increments. The comparison of solutions obtained for the velocity profile $f(\eta)$, which shows best agreement as depicted in Figure 2(d). As Figure 3(a) displays the micro-rotation profile rises, by increasing values of Gr , micro-rotation parameter. As we have inverse relation of Gr with viscosity, for larger values of Gr , viscosity lowers as a result the velocity of the fluid enhances. Figure 3(b) shows the change of the inertia parameter Nr on micro-rotation profile $g(\eta)$, for larger values of Nr , $g(\eta)$ reduces. The effects of the permeability parameter on non-dimensional micro-rotation profile $g(\eta)$ are shown in Figure 3(c), as the permeability parameter and viscosity of the fluid, is in inverse relation, so by increasing values of permeability parameter viscosity decreases as a result $g(\eta)$ increases. The comparison of solutions obtained for $g(\eta)$, shows the best fit as shown in Figure 3(d). It has been observed that temperature profile $\theta(\eta)$ declines for bigger values of radiation parameter R as shown in Figure 4(a), because the enhancement in radiations, drops the temperature $\theta(\eta)$ of the fluid. Figure 4(b) expresses the comparison of temperature profile $\theta(\eta)$ with the Prandtl number Pr , it shows a resemblance as a radiation parameter. The enrichment in Pr leads to a decrease in $\theta(\eta)$. From figure 4(c) it is clear that for lager values of Schmidt number Sc , $\theta(\eta)$ reduces, because the thickness in the boundary layer reduces. It is clear from Figure 4(d) that the temperature of the liquid diminishes for more prominent vales of Soret number Sr , is the proportion of temperature contrast and concentration contrast. Hence, the increase in the Soret number stands for an increase in $\theta(\eta)$. Figure 4(e) demonstrates that with the increment in Dufour number D_u , the temperature increases because specific heat increases as the thermal diffusion decreases. Both the solutions obtained for temperature profile $\theta(\eta)$ show the best correspondence as shown in Figure 4(f). The impact of Sr on concentration distribution $\phi(\eta)$ is shown in Figure 5(a), for greater values of Sr , the viscosity enhances as a result $\phi(\eta)$ rises. Figure 5(b) represents the impact of Schmidt number Sc on concentration profile $\phi(\eta)$, which shows that the variation in Sc enhances the concentration distribution, because the soret number is directly proportional to viscosity. It can be watched from Figure 5(c) that the non-dimensional concentration profile of the liquid rises with an increment of Dufour number D_u , which is the commitment of the concentration angle to the warm angle. The numerical solution for the concentration profile $\phi(\eta)$ shows the best agreement as demonstrated in Figure 5(d).

Table (1) shows the effects of radiations Δ , Prandtl number Pr , permeability Mr , inertia coefficient parameter Nr , Schmidt number Sc , Soret number Sr and Dufour number D_u on the skin contact C_f , Nusselt number Nu and Sherwood number Sh . It is seen from this table that the skin friction rate declines with increment in permeability parameter Mr . With the expanding values of Pr and radiation parameter Δ the of rate of heat transfer also increases. The rate of mass exchange increments with the increment of Schmidt number Sc and diminishes with Soret number Sr . Essentially with the expanding values of Dufour number D_u the rate of mass exchange increases.

Table 3: Numerical values of radial and tangential skin frictions, axial inflow, Nusselt number and magnetic skin depth for different values of the physical parameters

n	$f'(0)$	$-g'(0)$	$-h'(\infty)$	$-\theta'(0)$	$-m'(0)$
(a) For $\zeta = 0.5$, $\xi = 1$, $\alpha = 1.2$, $Re_m = 10$, $n = 1.5$, $Pr = 6.7$, $Re = 0.9$, $\epsilon = 0.3$, $R = 0.4$, $R_3 = 0.7$.					
A					
0.0	0.4448	0.6645	0.9612	0.7304	1.6101
0.5	-0.1153	1.0251	1.2959	1.0766	2.1755
0.0	-0.7976	1.2997	1.6234	1.2517	2.4390
(b) For $A = 0.09$, $\xi = 1$, $\alpha = 1.2$, $Re_m = 10$, $n = 1.5$, $Pr = 6.7$, $Re = 0.9$, $\epsilon = 0.3$, $R = 0.4$, $R_3 = 0.7$.					
ζ					
0.0	0.3922	0.7961	0.9403	0.9637	2.1024
0.5	0.3533	0.7369	1.0155	0.8117	1.7486
0.0	0.3167	0.6869	1.0949	0.6902	1.4841
(c) For $A = 0.09$, $\xi = 1$, $\alpha = 1.2$, $Re_m = 10$, $n = 1.5$, $Pr = 6.7$, $Re = 0.9$, $\epsilon = 0.3$, $R = 0.4$, $\zeta = 0.5$.					
R_3					
0.0	0.3807	0.6412	1.0343	0.8252	1.7857
0.5	0.3667	0.6899	1.0248	0.8183	1.7669
0.0	0.3248	0.8375	0.9951	0.7971	1.7090
(d) For $A = 0.09$, $\xi = 1$, $\alpha = 1.2$, $R_3 = 0.7$, $n = 1.5$, $Pr = 6.7$, $Re = 0.9$, $\epsilon = 0.3$, $R = 0.4$, $\zeta = 0.5$.					
Rem					
0.0	0.2602	0.8634	0.6957	0.7138	0.1010
0.5	0.2833	0.8352	0.8066	0.7433	0.3071
0.0	0.3005	0.8148	0.8801	0.7632	0.4831
(e) For $A = 0.09$, $\xi = 1$, $\alpha = 1.2$, $R_3 = 0.7$, $Rem = 10$, $Pr = 6.7$, $Re = 0.9$, $\epsilon = 0.3$, $R = 0.4$, $\zeta = 0.5$.					
n					
0.0	0.3800	0.7722	0.9562	0.8622	1.8771
0.5	0.3650	0.7522	0.9888	0.8338	1.8046
0.0	0.3576	0.7426	1.0054	0.8199	1.7694

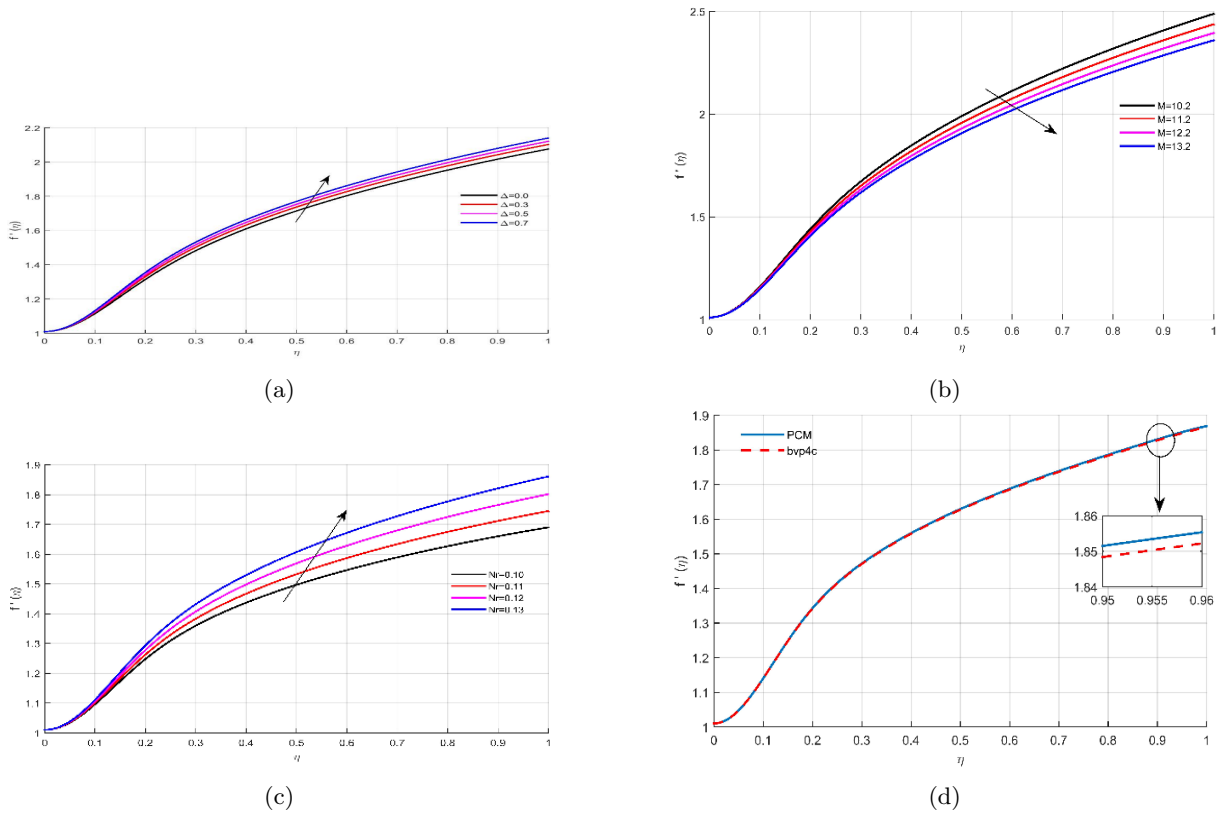


Figure 2: From (a)-(c), show the impact of Δ , Mr and Nr on non-dimensional velocity field $f'(\eta)$. (d). Comparison of solution obtained by PCM and bvp4c method. When $\Delta=0.3$, $R = 3.0$, $Mr = 10.2$, $Nr = 0.10$

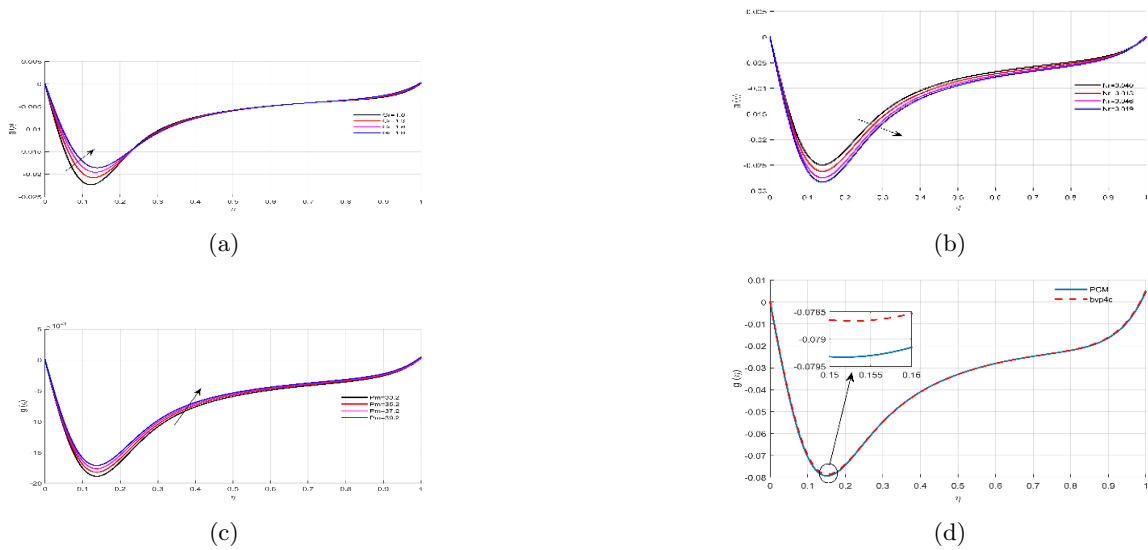


Figure 3: From (a)-(c), show the Micro rotation profile of $g(\eta)$ under the effect of Nr , Gr and Pm . (d). Comparison of solution obtained by PCM and bvp4c method. When $\Delta=0.3$, $R = 3.0$, $Mr = 10.2$, $Nr = 0.10$, $Gr = 1.0$, $Pr = 6.4$, $Sr = 1.00$, $Sc = 10.9$ and $D_u = 0.30$

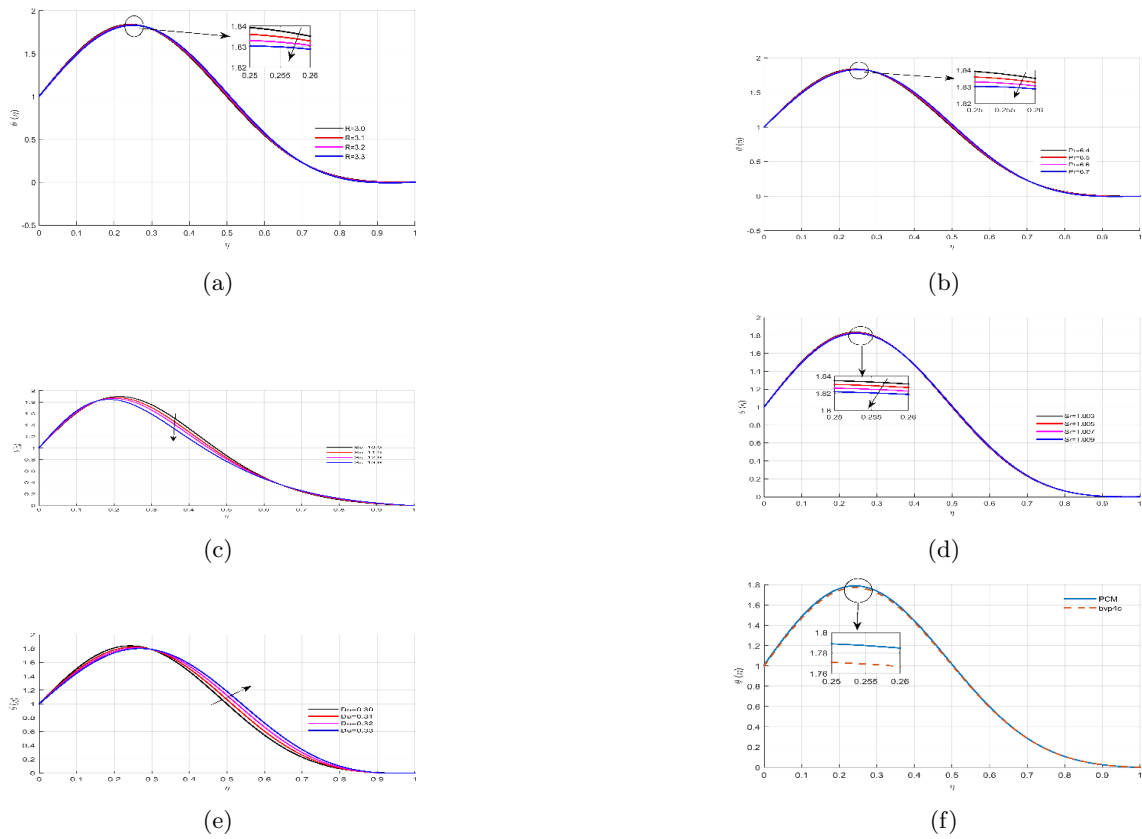


Figure 4: From (a)-(e), show the variation of dimensionless temperature profile of $\theta(\eta)$ under the effect of Rd , Pr , Sc , Sr and D_u . (f). Comparison of solution obtained by PCM and bvp4c method. When $\Delta=0.3$, $R = 3.0$, $Mr = 10.2$, $Nr = 0.10$, $Gr = 1.0$, $Pr = 6.4$, $Sr = 1.00$, $Sc = 10.9$ and $D_u = 0.30$

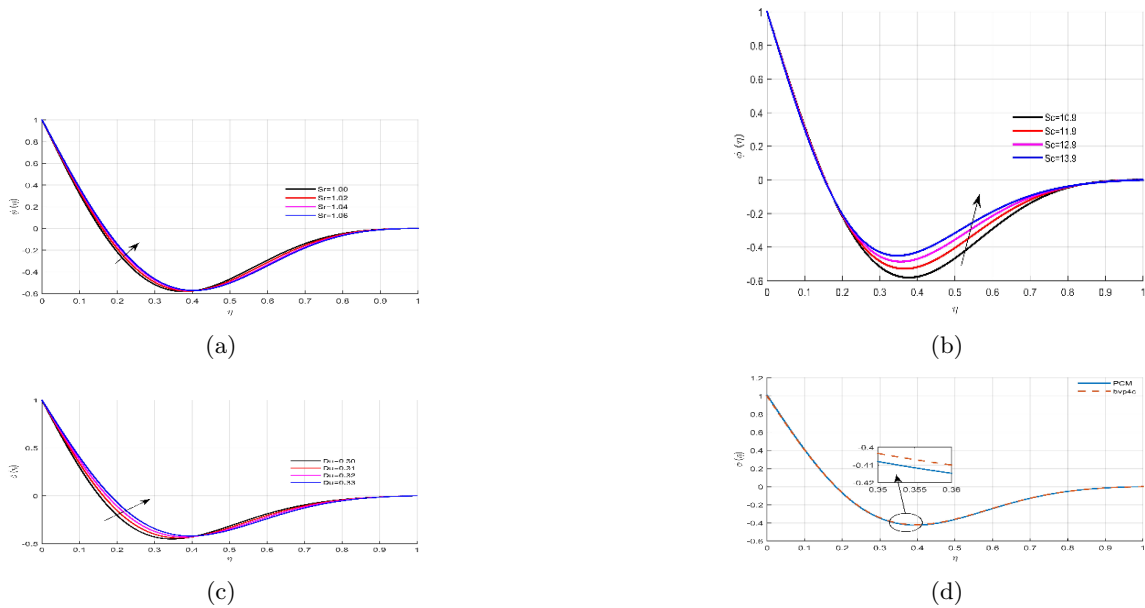


Figure 5: (a)-(c) The effects of parameters Sc , Sr and D_u on dimensional less concentration profile $\phi(\eta)$ respectively. (d) Comparison of solution obtained by PCM and bvp4c method. When $\Delta =0.3$, $R =3.0$, $Mr = 10.2$, $Nr =0 .10$, $Gr =1.0$, $Pr =6.4$, $Sr =1.00$, $Sc =10.9$ and $D_u =0.30$

4 Conclusion

We consider the stream of lean film, miniaturized scale polar liquid in permeable media past on moving lower plate with warm radiations beneath the impact of Soret, Schmidt, and Dofour impacts have been inspected. The system of non-linear coupled differential equations were solved through two numerical methods, PCM and bvp4c which shows best agreements and validity of our model. The impacts of imbedded parameters on the velocity, temperature, and concentration profiles are illustrated as well as examined. On the premise of our outcomes the following conclusion can be drawn.

1. The speed of the liquid increments with the improvement in vortex thickness parameter.
2. Permeability parameter declines the fluid velocity.
3. The fluid temperature declines with the increase in radiation parameter and Prandtl number.
4. The temperature profile increments whereas the concentration profile diminishes with expanding values of Schmidt number.
5. For three larger values of the radiation parameter and permeability parameter the coefficient of skin friction increases.
6. Enhancement in temperature and concentration has been observed with the increase in Dufour number.
7. The local mass exchange rate increment with the increments of the Schmidt number and Dufour number, and diminishes with the increment of Soret number.

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Conflicts of Interest The authors declare no conflict of interest.

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