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A scientific report on the flow of Maxwell fluid with heat transfer in vertical oscillating cylinder

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A B S T R A C T

The nonlinear nature of viscoelastic non-Newtonian fluids, introduce a unique challenge to physicists and mathematicians. By developing and utilizing viscoelastic models can play a special role in saving and treatments of every living species and to describe its particular characteristics. In the past three decades, viscoelastic fluid models are focused to improve its accuracy and reliability. Some rate type viscoelastic fluids include Maxwell fluid which effects in relaxation time. Such effect of relaxation time cannot be predicted by differential-type fluids. The polymers of low molecular weight are usefully described by Maxwell model. However, a keen interest of the researchers is seen in studying rate type fluids due to the fact that they incorporate both the elastic nature and memory behaviour together. In this article, viscoelastic Maxwell fluid is considered in cylindrical tube together with heat transfer due to convection caused by the buoyancy force. This problem is modelled using the classical approach and then solved for exact solution using joint the Laplace and Hankel transforms. Effects of pertinent parameters on Maxwell fluid velocity have been shown graphically. Behaviour of temperature is studied for various values of Prandtl number.

1. Introduction

In everyday life non-Newtonian fluids are widely used like lava, gums and blood in different fields of industries such as food industries, biomedicine and chemical engineering and many other industrial processes which make necessary for us to study non-Newtonian fluid flow behaviour. Due to its complex behaviour, various models in the literature are suggested for non-Newtonian fluids such as rate type [1], differential type [2] and integral type. Maxwell model is one subclass of rate type fluids. The polymers of less molecular weight are described best by Maxwell fluid model due to its relaxation in time effects and these effects of time relaxation cannot be seen in differential type fluids. The flow behaviour of rate type fluids can be seen in [3–5]. These fluids have non-direct connection between shear pressure and the rate of strain. Maxwell fluid in cylindrical shaped domain has got attention for the scholars, specialists and mathematicians dealing with the fluid mechanics. Many researchers of the fluid mechanics are interested to study the behaviour of viscoelastic Maxwell fluid in cylindrical domain [6, 7].

The book of Chanderashker [8] is considered basic for obtaining closed form solutions in cylindrical coordinates. Makris and Constantinou [9] discovered that it is not possible for Maxwell fluid to get sufficient experimental data due to range

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of its different frequencies. Friedrich [10] found that there is a relationship between governing equations involving fractional derivatives and molecular theories. Makris et al. [11] indicated that when classical Maxwell fluid is replaced by fractional Maxwell fluid best data regarding fluid are obtained. The constitutive equations of non-Newtonian and rate type fluids are studied by Rajagopal [12]. A lot of researchers contributed well to obtain a good research and to find exact solutions of the problems having shear stress boundary conditions. Water et al. [13] was the first to develop such kind of conditions Fetecau et al. [14] deal with viscoelastic fluid in cylindrical domain with unsteady shear stress and obtained its exact solutions. In another article Fetecau et al. [15] discussed the flow Burgers fluid due to oscillations inducted by cylinder. Vieru et al. [16] study Maxwell fluid motion due to oscillation of cylinder and obtained its starting solutions. The viscoelastic flow over sphere and the effect of inertia and flow behaviour due elasticity were studied by Zheng et al. [17]. A lot of researchers studied Maxwell fluid with heat transfer some of them are Zhao et al. [18] give their aptitude in heat transfer due to free convection boundary layer heat exchange of fractional Maxwell viscoelastic fluid over a vertical plate. Hernandez-Morales and Mitchell. [19] give basic idea that how to model fluid flow geomatery and mathematical form and study the heat and mass transfer in electroslag remelting process. Sheikholeslami et al. [20] deal about Lattice Boltzmann re-enactment of MHD free convection heat exchange of Al2O3– water nanofluid in a flat cylindrical shaped fenced in area with an internal triangular chamber. Zafar et al. [21] give their expertise in unsteady rotational flow of fractional Maxwell fluid in a cylinder subject to Shear stress on the boundary.

This paper studies convective flow of Maxwell fluid in a vertical cylinder. Closed form solutions are obtained by using Laplace and Hankel transforms jointly for momentum and energy. Effects of pertinent parameters on the velocity of Maxwell fluid have been shown graphically and discussed.

2. Mathematical Formulation and Solution of the Problem

We have taken the energy transfer and its effect on a Maxwell fluid flow through a vertical infinite cylinder of radius r_o. Radial coordinate r is taken perpendicular. It is suggested that at time $t \leq 0$, both the cylinder and fluid are in steady state with ambient temperature T_∞. At time $t = 0^+$, the cylinder begins to oscillate with velocity $U_o H(t)\exp(i\omega t)$ where U_o is the constant velocity, $H(t)$ is the Heaviside unit step function and ω denotes the oscillations frequency of the cylinder. At $t = 0^+$, the cylinder temperature rose to T_w which is then taken constant shown in Fig.1.

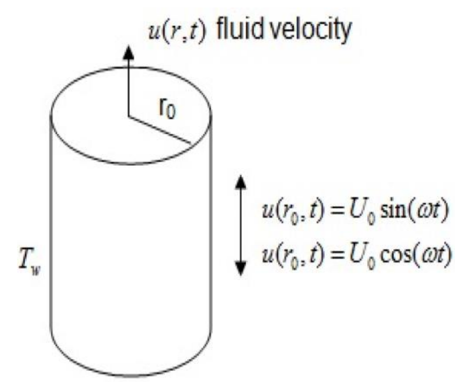

Fig.1 Fluid flow geometry

We consider that temperature and velocity and are the functions of radial coordinate and time such that the incompressibility constraint is satisfied identically. Under these assumptions, the governing equations can be modelled as [22]:

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial u(r,t)}{\partial t}=\nu\left(\frac{\partial^2 u(r,t)}{\partial r^2}+\frac{1}{r}\frac{\partial u(r,t)}{\partial r}\right)+\left(1+\lambda\frac{\partial}{\partial t}\right)g\beta_T(T-T_\infty);\quad r\in(0,r_\circ),\ t>0, \tag{1}$$

$$\frac{\partial^2 T(r,t)}{\partial r^2}+\frac{1}{r}\frac{\partial T(r,t)}{\partial r}-\frac{1}{\alpha}\frac{\partial T(r,t)}{\partial t}=0;\quad r\in(0,r_\circ),t>0, \tag{2}$$

With the following initial and boundary conditions:

$$u(r,0)=0,T(r,0)=T_\infty;\quad r\in\left[0,r_\circ\right], \tag{3}$$

$$u(r_\circ,t)=U_\circ H(t)e^{i\omega t};\quad T(r_\circ,t)=T_w,\quad t>0. \tag{4}$$

Presenting the accompanying dimensionless factors

$$t^*=\frac{t\nu}{r_\circ^2}\ ,\ r^*=\frac{r}{r_\circ}\ ,\ u^*=\frac{u}{U_\circ}\ ,\ \theta=\frac{T-T_\infty}{T_w-T_\infty}\ ,\ \omega^*=\frac{\omega r_\circ^2}{\nu},\lambda^*=\frac{\lambda\nu}{r_\circ^2}$$

Equations (1) - (4) are reduced to (dropping out star notation)

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\left(1+\lambda\frac{\partial}{\partial t}\right)Gr\theta;r\in(0,1),\ t>0, \tag{5}$$

$$\frac{\partial\theta(r,t)}{\partial t}=\frac{1}{\Pr}\left(\frac{\partial^2\theta(r,t)}{\partial r^2}+\frac{1}{r}\frac{\partial\theta(r,t)}{\partial r}\right);r\in(0,1),\ t>0, \tag{6}$$

$$u(r,0)=0\,,\theta(r,0)=0;\ r\in[0,1], \tag{7}$$

$$u(1,t)=H(t)\exp(i\omega t);\theta(1,t)=1,\quad t>0, \tag{8}$$

where $Gr=\frac{g\beta_T r_\circ^2(T_w-T_\infty)}{U_\circ\nu}$, $\Pr=\frac{\nu}{\alpha}$, $\alpha=\frac{k}{\rho c_p}$,

g is the acceleration because of gravity and β_T is the coefficient of heat expansion.

3. Calculation for Temperature

In order to solve the dimensionless system of above equations, initially, we apply the Laplace transform to Eqs. (6) And to the corresponding boundary condition from Eq. (8) To get the following transformed equations:

$$q\bar{\theta}(r,q)=\frac{1}{\Pr}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)\bar{\theta}(r,q), \tag{9}$$

$$\bar{\theta}(1,q)=\frac{1}{q}, \tag{10}$$

where $\bar{\theta}(r,q)$ is the Laplace transform for the function $\theta(r,t)$.

Now by applying the finite Hankel transform of zero order to Eq. (9), and using (10), we obtain:

$$\bar{\theta}_H\left(r_n,q\right)=\frac{J_1\left(r_n\right)}{r_n}\left[\frac{1}{q}-\frac{1}{\left[q+\frac{r_n^2}{\Pr}\right]}\right], \tag{11}$$

where $\bar{\theta}_H\left(r_n,q\right)=\int_0^1 r\,\bar{\theta}\left(r_n,q\right)J_0\left(rr_n\right)dr$ is the Hankel transform for the function $\bar{\theta}\left(r,q\right)$, and $J_0(.)$ is the Bessel function of order Zero and first kind and $r_n, n=0,1,......$ are the positive roots of the Eq. (11).

Inverse Laplace transform of Eq. (11), can be determined as:

$$\theta_H\left(r_n,t\right)=\frac{J_1\left(r_n\right)}{r_n}-\frac{J_1\left(r_n\right)}{r_n}\exp\left(-\frac{r_n^2}{\Pr}t\right), \tag{12}$$

Now apply the inverse Hankel transform to Eq. (12), we have:

$$\theta\left(r,t\right)=1-2\sum_{n=1}^{\infty}\frac{J_0\left(r_n r\right)}{r_n J_1\left(r_n\right)}\exp\left(-\frac{r_n^2}{\Pr}t\right). \tag{13}$$

The Nusselt number is determined in order to study the transformation of heat from the surface of the cylinder to the fluid and its dimensionless form can be evaluated as:

$$Nu=-\left(\frac{\partial\theta\left(r,t\right)}{\partial r}\right)_{r=1}=2\sum_{n=1}^{\infty}\exp\left(-\frac{r_n^2}{\Pr}t\right). \tag{14}$$

4. Calculation for velocity

Taking the Laplace transform of Eq. (6) and corresponding boundary condition from Eq. (9) take the form:

$$\left(1+\lambda q\right)q\bar{u}\left(r,q\right)=\frac{\partial^2\bar{u}\left(r,q\right)}{\partial r^2}+\frac{1}{r}\frac{\bar{u}\left(r,q\right)}{\partial r}+\left(1+\lambda q\right)Gr\bar{\theta}\left(r,q\right), \tag{15}$$

$$\bar{u}\left(1,q\right)=\frac{1}{q-i\omega}. \tag{16}$$

By using Eqs. (11) and (16), and applying the Hankel transform to Eq. (15) we obtain;

$$\bar{u}_H\left(r_n,q\right)=r_nJ_1\left(r_n\right)\frac{1}{\left(1+\lambda q\right)\left(q-i\omega\right)\left(q+\frac{r_n^2}{\left(1+\lambda q\right)}\right)}+Gr\frac{J_1\left(r_n\right)}{r_n}\frac{1}{\left(q+\frac{r_n^2}{\left(1+\lambda q\right)}\right)}\left[\frac{1}{q}-\frac{1}{\left[q+\frac{r_n^2}{\Pr}\right]}\right]. \tag{17}$$

Let we consider $\bar{u}_H\left(r_n,q\right)=F_n\left(q\right)+F_{3n}\left(q\right)$,

$$F_n\left(q\right)=r_nJ_1\left(r_n\right)\frac{1}{\left(1+\lambda q\right)\left(q-i\omega\right)\left(q+\frac{r_n^2}{\left(1+\lambda q\right)}\right)}=F_{1n}\left(q\right)+F_{2n}\left(q\right), \tag{18}$$

Where

$$F_{1n}(q)=\frac{J_1(r_n)}{r_n}\frac{1}{(q-i\omega)}-\frac{J_1(r_n)}{r_n}\frac{\omega(1+\lambda\iota\omega)(\omega+ir_n^2)}{(r_n^4+\omega^2(1+\lambda\iota\omega))}\frac{1}{(q-i\omega)}, \tag{19}$$

$$F_{2n}(q)=-\frac{r_nJ_1(r_n)(r_n^2-i\omega(1+\lambda\iota\omega))}{(q(1+\lambda\iota\omega)+r_n^2)(r_n^4+\omega^2(1+\lambda\iota\omega))}, \tag{20}$$

$$F_{3n}(q)=Gr\frac{J_1(r_n)}{r_n}\left[\frac{(1+\lambda q)}{r_n^2}\left(\frac{1}{q}-\frac{1}{\left(q+\frac{r_n^2}{(1+\lambda q)}\right)}\right)-\frac{\Pr}{r_n^2((1+\lambda q)-\Pr)}\left(\frac{1}{\left(q+\frac{r_n^2}{(1+\lambda q)}\right)}-\frac{1}{\left[q+\frac{r_n^2}{\Pr}\right]}\right)\right]. \tag{21}$$

Laplace inversion of the above equations yields

$$F_n(t)=f_{1n}(t)+f_{2n}(t), \tag{22}$$

With

$$f_{1n}(t)=\frac{J_1(r_n)e^{i\omega t}}{r_n}-\frac{J_1(r_n)\omega(1+\lambda\iota\omega)}{r_n((1+\lambda\iota\omega)\omega^2+r_n^4)}\begin{pmatrix}\omega\cos(\omega t)\\ -r_n^2\sin(\omega t)\end{pmatrix}-i\frac{J_1(r_n)\omega(1+\lambda\iota\omega)}{r_n((1+\lambda\iota\omega)\omega^2+r_n^4)}\begin{pmatrix}r_n^2\cos(\omega t)\\ +\omega\sin(\omega t)\end{pmatrix}, \tag{23}$$

$$f_{2n}(t)=\frac{r_nJ_1(r_n)(1-\lambda^2\omega^2)\exp(-r_n^2)t}{(r_n^4+(1+\lambda\iota\omega)\omega^2)}\left[i\omega(1+\lambda\iota\omega)-r_n^3\right], \tag{24}$$

$$f_{3n}(t)=\frac{GrJ_1(r_n)}{(r_n)}\left(\frac{t-\lambda^2}{\lambda t}\right)\frac{2\exp\left(-\frac{2}{\lambda}\right)t}{\sqrt{\left(\frac{1}{\lambda}\right)^2-4\left(\frac{r_n^2}{\lambda}\right)}}\sinh\left(\frac{\sqrt{\left(\frac{1}{\lambda}\right)^2-4\left(\frac{r_n^2}{\lambda}\right)}t}{2}\right)\left[1+\left(\frac{1-\lambda}{r_n^2}\right)-\frac{Pr}{r_n^2\lambda\left(\frac{1}{\lambda}-\frac{Pr}{r_n^2\lambda}+1\right)}\right], \tag{25}$$

Using Eqn. (22)-(25) we obtain;

$$\begin{aligned}u_H(r_n,t)&=\frac{J_1(r_n)}{r_n}\exp(i\omega t)-\frac{J_1(r_n)\omega(1+\lambda\iota\omega)}{r_n((1+\lambda\iota\omega)\omega^2+r_n^4)}(\omega\cos(\omega t)-r_n^2\sin(\omega t))\\ &-i\frac{J_1(r_n)\omega(1+\lambda\iota\omega)}{r_n((1+\lambda\iota\omega)\omega^2+r_n^4)}(r_n^2\cos(\omega t)+\omega\sin(\omega t))-\frac{r_nJ_1(r_n)(1-\lambda^2\omega^2)\exp(-r_n^2)t}{(r_n^4+(1+\lambda\iota\omega)\omega^2)}\left[i\omega(1+\lambda\iota\omega)-r_n^3\right]+\\ &\frac{GrJ_1(r_n)}{(r_n)}\left(\frac{t-\lambda^2}{\lambda t}\right)\frac{2\exp\left(-\frac{2}{\lambda}\right)t}{\sqrt{\left(\frac{1}{\lambda}\right)^2-4\left(\frac{r_n^2}{\lambda}\right)}}\sinh\left(\frac{\sqrt{\left(\frac{1}{\lambda}\right)^2-4\left(\frac{r_n^2}{\lambda}\right)}t}{2}\right)\left[1+\left(\frac{1-\lambda}{r_n^2}\right)-\frac{Pr}{r_n^2\lambda\left(\frac{1}{\lambda}-\frac{Pr}{r_n^2\lambda}+1\right)}\right]\end{aligned} \tag{26}$$

Now by taking inverse Hankel transform, Eq. (26) reduces to the following form:

$$u(r,t)=\exp(i\omega t)-\begin{bmatrix} 2\sum_{n=1}^{\infty}\frac{J_o(rr_n)}{J_1(r_n)}\frac{(1+\lambda\iota\omega)\,\omega^2\cos(\omega t)}{r_n\left((1+\lambda\iota\omega)\,\omega^2+r_n^4\right)}+ \\ 2\sum_{n=1}^{\infty}\frac{J_o(rr_n)}{J_1(r_n)}\frac{\omega(1+\lambda\iota\omega)\,r_n}{\left((1+\lambda\iota\omega)\,\omega^2+r_n^4\right)} \end{bmatrix}-\iota\left[2\sum_{n=1}^{\infty}\frac{J_o(rr_n)}{J_1(r_n)}\frac{r_n}{\left((1+\lambda\iota\omega)\,\omega^2+r_n^4\right)}+2\sum_{n=1}^{\infty}\frac{J_o(rr_n)}{J_1(r_n)}\frac{(1+\lambda\iota\omega)\,\omega^2\sin(\omega t)}{r_n\left((1+\lambda\iota\omega)\,\omega^2+r_n^4\right)}\right]$$

$$-2\sum_{n=1}^{\infty}\frac{J_o(rr_n)}{J_1(r_n)}\frac{r_n^{\,3}}{\left((1+\lambda\iota\omega)\,\omega^2+r_n^4\right)}(1+\lambda\iota\omega)\exp\left(\frac{-r_n^2}{(1+\lambda\iota\omega)}\right)t+2\sum_{n=1}^{\infty}\frac{J_o(rr_n)}{J_1(r_n)}\frac{r_n\iota\omega(1+\lambda\iota\omega)}{\left((1+\lambda\iota\omega)\,\omega^2+r_n^4\right)}(1+\lambda\iota\omega)\exp\left(\frac{-r_n^2}{(1+\lambda\iota\omega)}\right)t$$

$$+2Gr\sum_{n=1}^{\infty}\frac{J_o(rr_n)}{J_1(r_n)}\left(\frac{t-\lambda^2}{r_n\lambda t}\right)\frac{2\exp\left(-\frac{2}{\lambda}\right)t}{\sqrt{\left(\frac{1}{\lambda}\right)^2-4\left(\frac{r_n^2}{\lambda}\right)}}\sinh\left(\frac{\sqrt{\left(\frac{1}{\lambda}\right)^2-4\left(\frac{r_n^2}{\lambda}\right)}\,t}{2}\right)\begin{bmatrix}1+\left(\frac{1-\lambda}{r_n^3}\right) \\ -\frac{Pr}{r_n^2\lambda\left(\frac{1}{\lambda}-\frac{Pr}{r_n^2\lambda}+1\right)}\end{bmatrix}. \tag{27}$$

5. Graphical results and discussion

The impact of different fluid parameters on temperature $\theta(r,t)$, fluid velocity $u(r,t)$ has been discussed graphically by using the computational tool Mathcad. The fluid flow geometry has been shown in figure 1. In figures 2 Grashoff number effect has been shown on the Maxwell fluid parameter. By increasing the Grashoff number both the velocities increases due to the increase in the buoyancy term because Grashoff number represents the ratio between the buoyancy force and viscous force. In figure 3 the effect of Prandtl number has been noticed on the viscoelastic fluid. As Prandtl number is the ratio of viscous forces to the thermal forces. By increases Prandtl number the viscous forces becomes dominant as a result fluid velocity decreases. . In figure 4 the influence of Prandtl number is noticed. The curves of temperature are sketched versus r. It is found that an increase in Prandtl number decreases the fluid temperature due to increase in the viscous diffusion rate and a decrease in thermal diffusion rate. Variation of Nusselt number for various standards of Pr is studied in figure 5. As Nusselt number is the ratio between convective and conductive heat transfer. It is indicated that by increasing Pr, downfall occur in the Nusselt number due to the variation in viscosity rate so for less value of Pr, convection is dominant and for high values conduction become dominant. In figure 6 comparative studies has been made for viscous fluids and viscoelastic Maxwell fluid for both sine and cosine oscillation. In fig 7 limiting solution has been plotted, it is indicated that by putting λ =0 will reduce our solution to solution obtained by Naheed et al. [23].

6. CONCLUSION

Some rate type viscoelastic fluids include Maxwell fluid which effects in relaxation time. Such effect of relaxation time cannot be predicted by differential-type fluids. The polymers of low molecular weight are usefully described by Maxwell model. In this article, viscoelastic Maxwell fluid is considered in cylindrical tube together with heat transfer due to convection caused by the buoyancy force. This problem is modelled using the classical approach and then solved for exact solution using joint the Laplace and Hankel transforms. Effects of pertinent parameters on Maxwell fluid velocity have been shown graphically. Behaviour of temperature is studied for various values of Prandtl number. The key points of the present study are listed as under:

1. Velocity decrease with increase in Prandtl number due to increase in viscous diffusion rate.

2. Velocity increases with increase in Grashof number due to increase in buoyancy force.

3. Velocity decreases due to increase in Maxwell parameter λ due to relaxation in time effects which causes increase in the viscoelasticity of the fluid. The phenomena are absorbed because less relaxation time has low elasticity and larger relaxation time indicates higher and lesser recovery ability of Maxwell fluid.

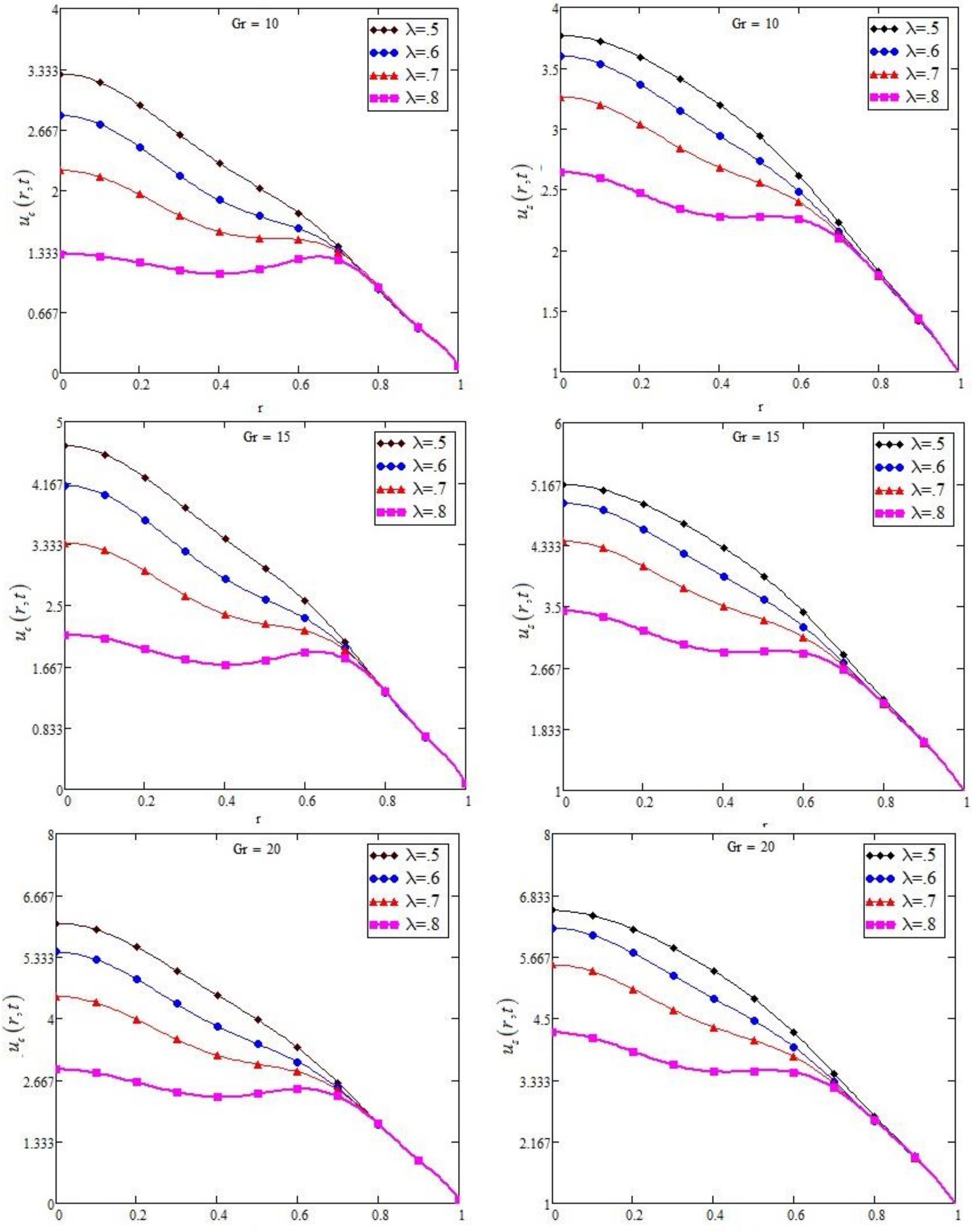

Fig.2 Graphs of velocity in cosine and sine oscillations for variation in Gr. when $\Pr = 1, t = 2.5, \omega = 0.6$

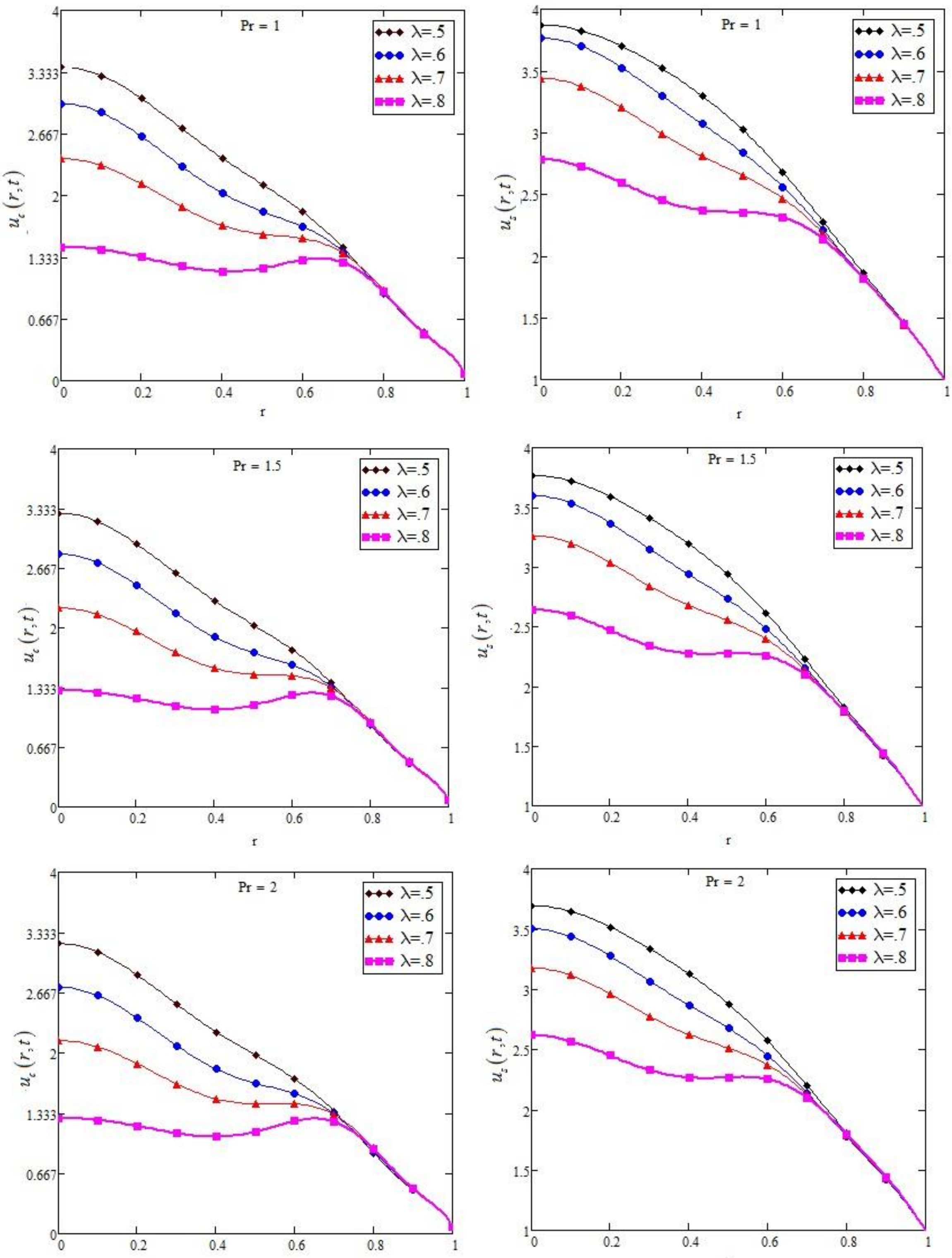

Fig.3 Graphs of velocity for cosine and sine oscillations for variation in Pr when $Gr = 10$

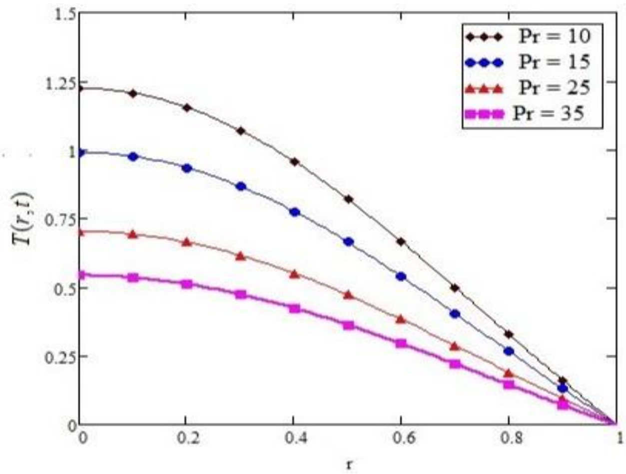

Fig.4 Graphs of temperature for different values of Pr when $t = 2.5, \omega = .6$

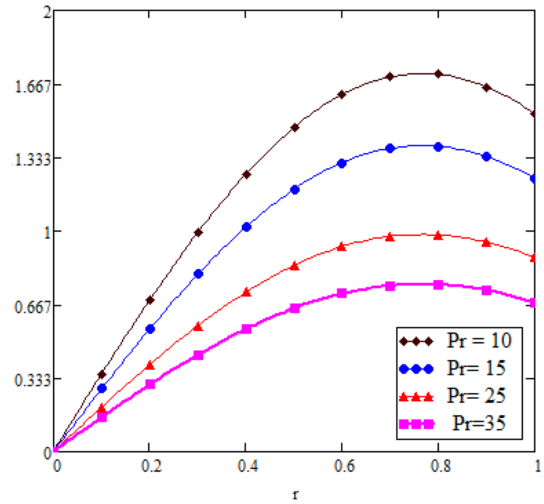

Fig.5 Variation in Nusselt number for different values of Pr when $t = 2.5, \omega = .6$

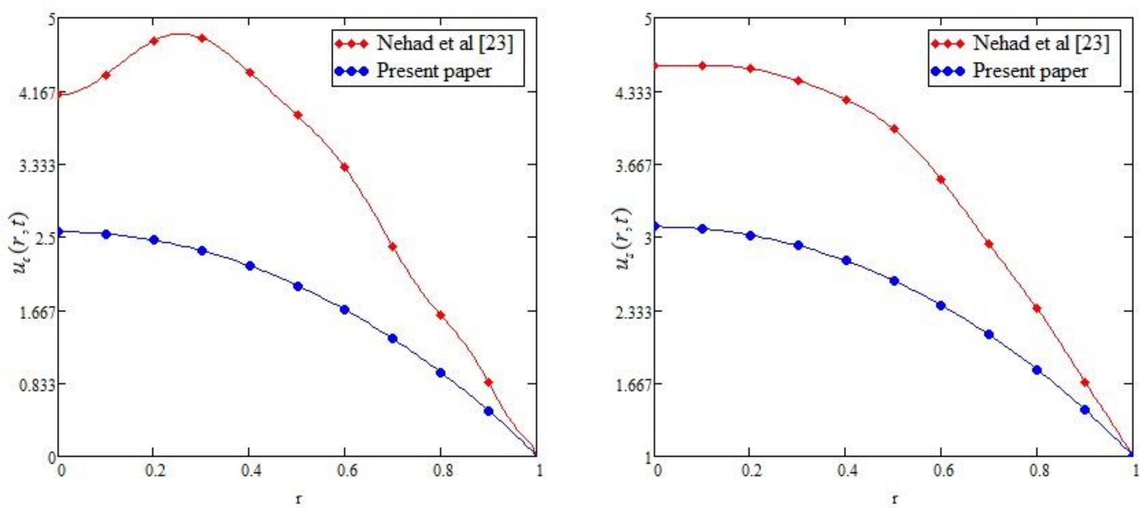

Fig.6 Comparative graph of viscous fluid and Maxwell fluid for sine and cosine oscillation.

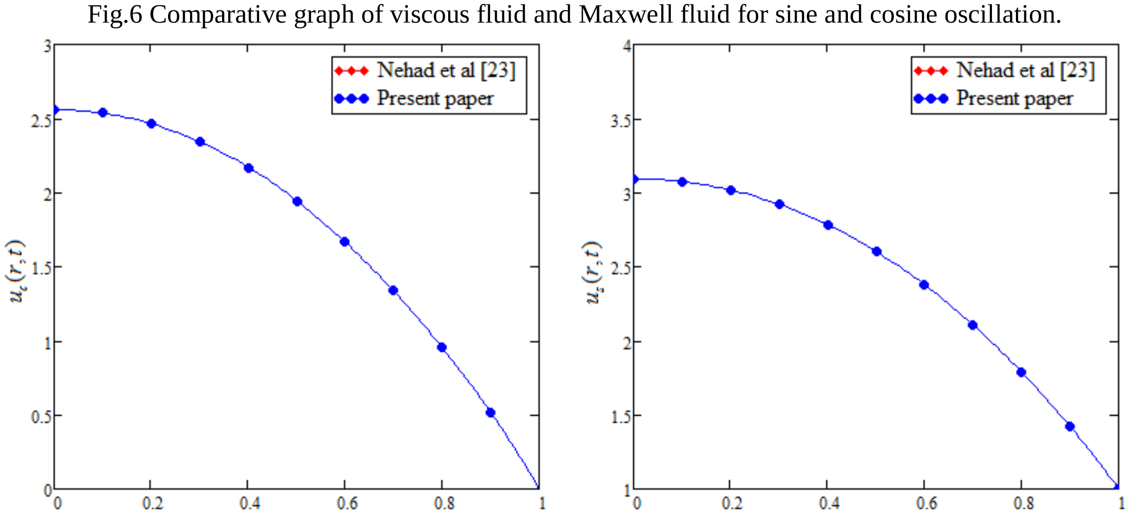

Fig.7 Limiting solution for sine and cosine oscillation by putting $\lambda = 0$.

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